

11-2 Operations With Radical Expressions Day 84

p. 622 Textbook

Check Skills You'll Need (1-7)

Simplify each radical expression.

1. $\sqrt{52}$
 $\frac{\sqrt{4 \cdot 13}}{2\sqrt{13}}$

3. $4\sqrt{54}$
 $4 \cdot 3\sqrt{6}$
 $12\sqrt{6}$

2. $\sqrt{200}$
 $\frac{\sqrt{100 \cdot 2}}{10\sqrt{2}}$

4. $\sqrt{125x^2}$
 $\sqrt{25 \cdot 5 \cdot x^2}$
 $5x\sqrt{5}$

4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196

Rationalize each denominator.

5. $\frac{\sqrt{3}}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}} = \frac{\sqrt{33}}{\sqrt{121}} = \frac{\sqrt{33}}{11}$

6. $\frac{\sqrt{5}}{\sqrt{8}} \cdot \frac{\sqrt{4 \cdot 2}}{\sqrt{4 \cdot 2}} = \frac{\sqrt{5} \cdot \sqrt{2}}{2\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{10}}{2 \cdot 2} = \frac{\sqrt{10}}{4}$

7. $\frac{\sqrt{15}}{\sqrt{2x}} \cdot \frac{\sqrt{2x}}{\sqrt{2x}} = \frac{\sqrt{30x}}{\sqrt{4x^2}} = \frac{\sqrt{30x}}{2x}$

1 EXAMPLE Combining Like Radicals

Simplify $\sqrt{2} + 3\sqrt{2}$.

When adding coefficients or subtracting the Radicands must be the same. Then add/sub. the coefficients and the radicand stays the same.

$1 + 3$
 $4\sqrt{2}$

Quick Check Simplify each expression.

a. $-3\sqrt{5} - 4\sqrt{5}$
 $-7\sqrt{5}$

b. $\sqrt{10} - 5\sqrt{10}$
 $\frac{1\sqrt{10}}{-5\sqrt{10}} \rightarrow -4\sqrt{10}$

2 EXAMPLE Simplifying to Combine Like Radicals

Simplify $7\sqrt{3} - \sqrt{12}$.

$$\begin{aligned} & \sqrt{4 \cdot 3} \\ & \downarrow \\ 7\sqrt{3} - 2\sqrt{3} \\ & \underline{5\sqrt{3}} \end{aligned}$$

Quick Check 2 Simplify each expression.

a. $3\sqrt{20} + 2\sqrt{5}$

$$\begin{aligned} & 3 \cdot \sqrt{4 \cdot 5} \\ & 3 \cdot 2\sqrt{5} \\ & 6\sqrt{5} + 2\sqrt{5} = 8\sqrt{5} \end{aligned}$$

b. $3\sqrt{3} - 2\sqrt{27}$

$$\begin{aligned} & 3\sqrt{3} - 2\sqrt{9 \cdot 3} \\ & 3\sqrt{3} - 2 \cdot 3\sqrt{3} \\ & 3\sqrt{3} - 6\sqrt{3} \\ & \underline{-3\sqrt{3}} \end{aligned}$$

3 EXAMPLE Using the Distributive Property

Simplify $\sqrt{3}(\sqrt{6} + 7)$.

$$\begin{aligned} & \sqrt{3} \cdot \sqrt{6} + \sqrt{3} \cdot 7 \\ & \sqrt{18} + 7\sqrt{3} \\ & \sqrt{9 \cdot 2} \\ & \underline{3\sqrt{2} + 7\sqrt{3}} \end{aligned}$$

Quick Check 3 Simplify each radical expression.

a. $\sqrt{5}(2 + \sqrt{10})$

b. $\sqrt{2x}(\sqrt{6x} - 11)$

$$\begin{aligned} & 2\sqrt{5} + \sqrt{50} \\ & \quad \quad \quad \sqrt{25 \cdot 2} \\ & \underline{2\sqrt{5} + 5\sqrt{2}} \end{aligned} \quad \left\{ \begin{aligned} & \sqrt{2x} \cdot \sqrt{6x} - \sqrt{2x} \cdot 11 \\ & \sqrt{12x^2} - 11\sqrt{2x} \\ & \sqrt{4 \cdot 3 \cdot x^2} \\ & \underline{2x\sqrt{3} - 11\sqrt{2x}} \end{aligned} \right.$$

c. $\sqrt{5a}(\sqrt{5a} + 3)$

$$\begin{aligned} & \sqrt{25a^2} + 3\sqrt{5a} \\ & \underline{5a + 3\sqrt{5a}} \end{aligned}$$

4 EXAMPLE Simplifying Using FOIL

Simplify $(\sqrt{5} - 2\sqrt{15})(\sqrt{5} + \sqrt{15})$.

$$\begin{array}{r} \sqrt{5} \cdot \sqrt{5} + \sqrt{5} \sqrt{15} - 2\sqrt{15} \cdot \sqrt{5} - 2\sqrt{15} \cdot \sqrt{15} \\ \sqrt{25} + \sqrt{75} - 2\sqrt{75} - 2\sqrt{225} \\ \sqrt{25} \cdot \sqrt{3} \quad \sqrt{25} \cdot \sqrt{3} \quad - 2\sqrt{25} \cdot \sqrt{3} \quad - 2\sqrt{25} \cdot \sqrt{3} \\ \hline 5 + 5\sqrt{3} - 2 \cdot 5\sqrt{3} - 2 \cdot 15 \\ \hline -25 - 5\sqrt{3} \end{array}$$

Quick Check 4 Simplify each radical expression.

a. $(2\sqrt{6} + 3\sqrt{3})(\sqrt{6} - 5\sqrt{3})$

$$\begin{array}{r} 2\sqrt{36} - 10\sqrt{18} + 3\sqrt{18} - 15\sqrt{9} \\ 2 \cdot 6 - 7\sqrt{18} - 15 \cdot 3 \\ 12 - 7 \cdot 3\sqrt{2} - 45 \\ -33 - 21\sqrt{2} \end{array}$$

b. $(\sqrt{7} + 4)^2$

$$\begin{array}{r} (\sqrt{7} + 4)(\sqrt{7} + 4) \\ \sqrt{7} \cdot \sqrt{7} + 4 \cdot \sqrt{7} + 4 \cdot \sqrt{7} + 4 \cdot 4 \\ \sqrt{49} + 8\sqrt{7} + 16 \\ 7 + 8\sqrt{7} + 16 \\ 8\sqrt{7} + 23 \end{array}$$

Conjugates are the sum and the difference of the same two terms. The radical expressions $\sqrt{5} + \sqrt{2}$ and $\sqrt{5} - \sqrt{2}$ are conjugates. The product of two conjugates results in a difference of two squares.

$$\begin{aligned} (\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2}) &= (\sqrt{5})^2 - (\sqrt{2})^2 \\ &= 5 - 2 \\ &= 3 \end{aligned}$$

Notice that the product of these conjugates has no radical.