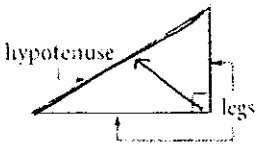
3,9 Day 85

1 Solving Problems Using the Pythagorean Theorem

In a right triangle, the side opposite the right angle is the **hypotenuse.** It is the longest side. Each of the sides forming the right angle is a **leg.**



The **Pythagorean Theorem** describes the relationship of the lengths of the sides of a right triangle.

Theorem The Pythagorean Theorem

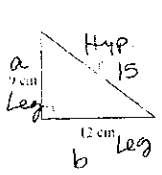
In any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$a^2 + b^2 = c^2$

Hypotenuse, a Leg, b Leg

$Leg^2 + Leg^2 = Hypo^2$

1 EXAMPLE Using the Pythagorean Theorem



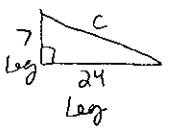
$9^2 + 12^2 = c^2$

$81 + 44 = c^2$

$\sqrt{225} = \sqrt{c^2}$

$15 = c$

❶ What is the length of the hypotenuse of a right triangle with legs of lengths 7 cm and 24 cm?



$7^2 + 24^2 = c^2$

$49 + 576 = c^2$

$\sqrt{625} = \sqrt{c^2}$

$25 = c$

Pythagorean Triples

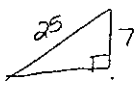
9, 12, 15

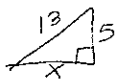
*7, 24, 25

*3, 4, 5

6, 8, 10

*5, 12, 13





$5^2 + x^2 = 13^2$

$25 + x^2 = 169$

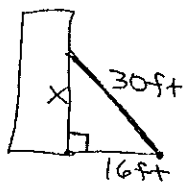
$-25 \quad -25$

$\sqrt{x^2} = \sqrt{144}$

$x = 12$

2 EXAMPLE Real-World Problem Solving

Fire Rescue A fire truck parks beside a building such that the base of the ladder is 16 ft from the building. The fire truck extends its ladder 30 ft as shown at the left. How high is the top of the ladder above the ground?



$x^2 + 16^2 = 30^2$

$x^2 + 256 = 900$

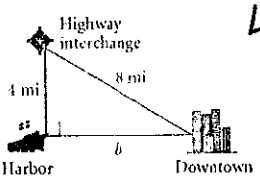
$-256 \quad -256$

$\sqrt{x^2} = \sqrt{644}$

$x = 25.37$

25.4ft

❷ Use the figure at the right. About how many miles is it from downtown to the harbor? Round to the nearest tenth of a mile.



$4^2 + b^2 = 8^2$

$16 + b^2 = 64$

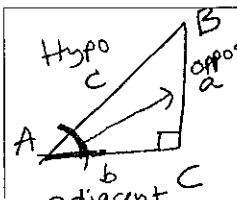
$-16 \quad -16$

$\sqrt{b^2} = \sqrt{48}$

$\sqrt{48} = 4\sqrt{3}$

$\sqrt{16}\sqrt{3}$

OR $b \approx 6.9$ mi



Use TRIG (11.5) to Find missing sides & missing acute ∠'s OF right Δ's.

acute ∠'s

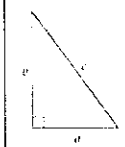
Sine A $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$

cosine A $\cos A = \frac{\text{adjacent}}{\text{hypo.}}$

tangent A $\tan A = \frac{\text{opposite}}{\text{adjacent}}$

SOH CAH TOA

Use the triangle at the right. Find the length of the missing side. If necessary, round to the nearest tenth.



① $6^2+8^2=c^2$

$36+64=c^2$

$\sqrt{100}=\sqrt{c^2}$

$10=c$

1. $a = 6, b = 8$
2. $a = 15, b = 20$
3. $a = 8, b = 15$
4. $a = 10, b = 24$
5. $a = 1.5, b = 2$
6. $a = \frac{3}{5}, b = \frac{4}{5}$
7. $a = 3, c = 5$
8. $b = 12, c = 13$
9. $a = 9, c = 15$
10. $b = 7, c = 10$
11. $a = 5, c = 9$
12. $a = 0.8, c = 1$

⑦ $3^2+b^2=5^2$

$9+b^2=25$

$-9 \quad -9$

$\sqrt{b^2}=\sqrt{16}$

$b=4$

sine

$\sin \angle A = \frac{\text{opposite}}{\text{hypotenuse}}$

cosine

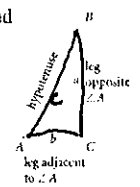
$\cos \angle A = \frac{\text{adjacent}}{\text{hypotenuse}}$

tangent

$\tan \angle A = \frac{\text{opposite}}{\text{adjacent}}$

SOH CAH TOA

Ratios of the sides of a right triangle are called **trigonometric ratios.** In $\triangle ABC$, at the right, you see the relationships between an angle and the legs of a triangle. The same letter indicates the length of a side of the triangle and the angle opposite that side.



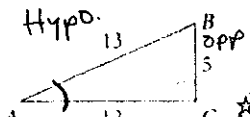
sine of $\angle A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}}$ or $\sin A = \frac{a}{c} = \frac{\text{opposite leg}}{\text{hypotenuse}}$

cosine of $\angle A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}}$ or $\cos A = \frac{b}{c} = \frac{\text{adjacent leg}}{\text{hypotenuse}}$

tangent of $\angle A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A}$ or $\tan A = \frac{a}{b} = \frac{\text{opposite leg}}{\text{adjacent leg}}$

1 EXAMPLE **Finding Trigonometric Ratios**

Use the triangle below. Find sin A, cos A, and tan A.

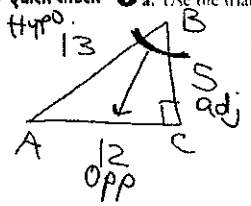


$\sin A = \frac{5}{13}$ $\frac{O}{H}$

$\cos A = \frac{12}{13}$ $\frac{A}{H}$

$\tan A = \frac{5}{12}$ $\frac{O}{A}$

Quick Check **1 a.** Use the triangle in Example 1. Find sin B, cos B, and tan B.



$\sin B = \frac{12}{13}$ $\frac{O}{H}$

$\cos B = \frac{5}{13}$ $\frac{A}{H}$

$\tan B = \frac{12}{5}$ $\frac{O}{A}$

b. Critical Thinking What is the relationship between sin A and cos B?

mode must be in degrees on the calculator.

You can use a calculator to find the values of the trigonometric ratios when you know the measure of an angle.

2 Find the value of each expression. Round to the nearest ten-thousandth.

a. sin 70° .9397 b. cos 70° .3420

c. tan 70° 2.7475 d. sin 5° = .0872

$\sin 70^\circ = .939692\ldots$

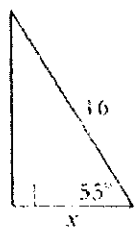
.12345

tenths, hundredths, thousandths, ten-thousandths

You can use trigonometry to find missing lengths in a right triangle when you know the length of one side and the measure of one of the angles.

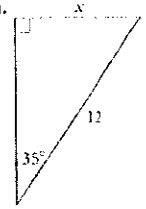
3 EXAMPLE **Finding Missing Side Lengths**

Find the value of x in the triangle at the right.

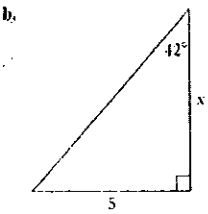


3 Find the value of x in each triangle. Round to the nearest tenth.

a.

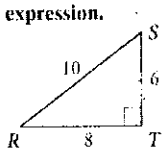


b.



Use $\triangle RST$ at the right. Find the value of each expression.

1. sin R
2. cos R
3. tan R
4. sin S
5. cos S
6. tan S



$\sin R = \frac{6}{10} = \frac{3}{5}$ Reduce!

$\cos R = \frac{8}{10} = \frac{4}{5}$

$\tan R = \frac{6}{8} = \frac{3}{4}$

$\sin S = \frac{8}{10} = \frac{4}{5}$

$\cos S = \frac{6}{10} = \frac{3}{5}$

$\tan S = \frac{8}{6} = \frac{4}{3}$

Find the value of each expression.
Round to the nearest ten-thousandth.

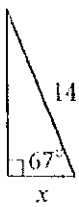
7. sin 32°
8. cos 55°
9. tan 52°
11. cos 15°
12. tan 87°
13. cos 30°

(7) .5299 (8) .5736

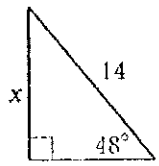
(9) 1.2799 (11) .9659

Find the value of x to the nearest tenth.

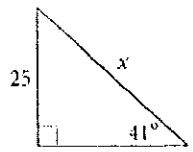
15.



16.



18.



19.