Lesson 13: Probability

Chapter 10

Probability (P) is a measure of the likelihood that an event will occur. (In this lesson, when you are asked to find the probability of choosing a certain item from a container, assume all trials are done without looking.)

10.3

Sample Space

A **sample space** (S) is the set { } of all possible outcomes of an event. (Duplicate outcomes are omitted.)

Example

Bruce wandered off the hiking trail and got lost in the woods. He needs to decide which direction he should travel to get back to the trail. What is the sample space?

The sample space consists of the four directions.

S = {north, east, south, west}

Example

The letters of the word "mathematics" are written on slips of paper and put into a hat. A slip of paper is drawn and the letter on the slip is recorded. What is the sample space?

The sample space consists of the letters of the word "mathematics." There are eight outcomes. (Duplicate letters are omitted.)

S = {M, A, T, H, E, I, C, S}

↑ listed M, A, T once

Practice

Directions: For Numbers 1 through 3, write the sample space.

1. The letters of the word "Ohio": S = {O, H, I}

only write "O" once

2. The even whole numbers less than 20: S = {0, 2, 4, 6, 8, 10, 12, 14, 16, 18}

$n < 20$

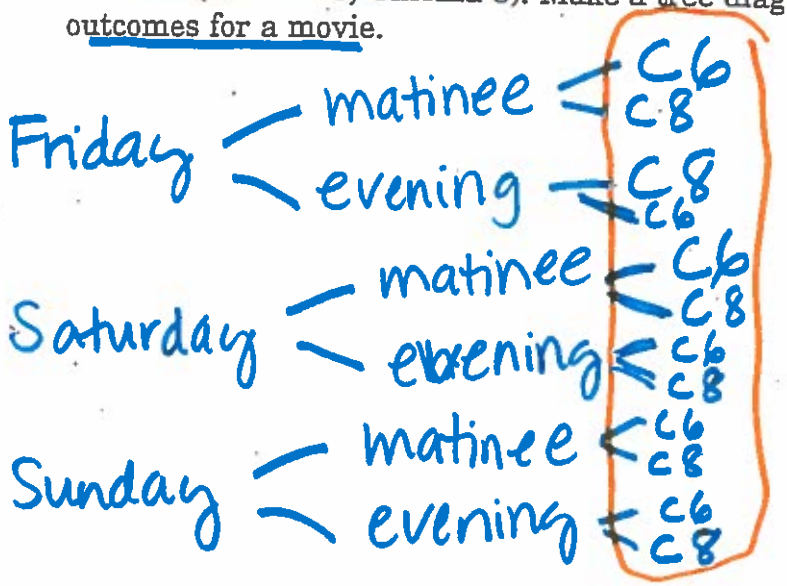
3. The months of the year that have exactly 30 days:

S = {June, September, November, April}

Practice

Tree Diagrams

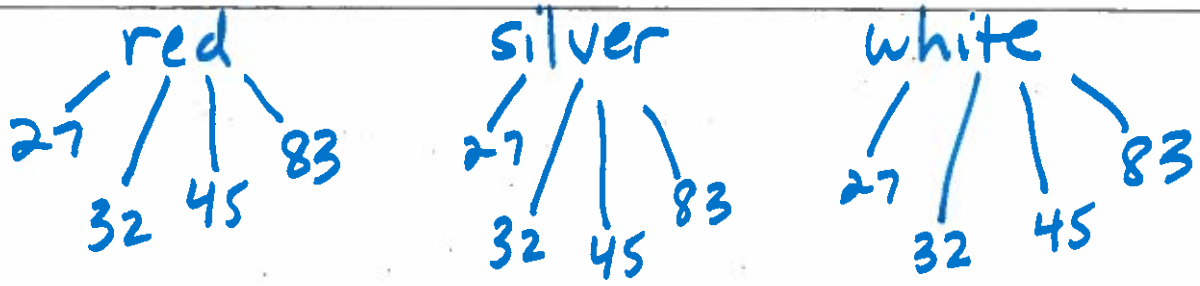
1. Maren and Melissa are going to see a movie. They can choose from three days (Friday, Saturday, Sunday), two times (matinee, evening), and two theaters (Cinema 6, Cinema 8). Make a tree diagram to show the possible outcomes for a movie.



number of outcomes: 12

days · times · cinemas = 12
3 · 2 · 2

2. A sporting goods store sells football jerseys that come in three colors (red, silver, white) with four possible numbers (27, 32, 45, 83). Make a tree diagram to show the possible outcomes for a jersey.



number of outcomes: 12

textbook P.448 (9-24)

3 colors · 4 numbers = 12

Find the # of possible outcomes by multiplying the # of outcomes for each event.

The Basic Counting Principle Day 60

Skill: the basic counting principle

Name _____

In Eureka, NV, it is against the law for a man to habitually kiss others if he has what?

To find out, use the basic counting principle to find the answers to the following questions. Find the answers at the bottom of the page and put the corresponding letter above each answer.

GOOD CHOW DINER MENU

H. pizza crust: wheat, white (2 · 7)
toppings: pepperoni, onion, mushrooms, sausage, olives, hamburger, green pepper
How many different one-topping pizzas are possible? 14

U. tortilla: flour, corn, blue corn
fillings: guacamole, bean, beef, chicken, cheese, chili
How many different one-filling burritos are possible? $3 \cdot 6 = 18$

S. main dishes: hamburger, hot dog, grilled cheese, meat loaf
side dishes: mashed potatoes, cole slaw, French fries, cottage cheese
How many different meals are possible? $4 \cdot 4 = 16$

M. soups: French onion, clam chowder, chicken noodle, split pea, vegetable
salads: house, Caesar, chef, cobb, pasta
How many different soup and salad combinations are possible?

A. bread: rye, wheat, white, French, sourdough, pumpernickel
fillings: corned beef, ham, chicken salad, roast beef, salami
How many different sandwiches are possible?

A. pasta: penne, angel hair, linguini, gnachi, shells
sauce: marinara, alfredo, clam sauce
How many different pasta and sauce combinations are possible? $5 \cdot 3 = 15$

C. bagels: egg, plain, onion, garlic
cream cheese: plain, vegetable, pineapple, chive, strawberry, lox $4 \cdot 6 = 24$
How many different bagel and cream cheese combinations are possible?

E. omelettes: Californian, Western, Spanish, Cheese Lover $4 \cdot 5 = 20$
fruit: grapefruit, canteloupe, bananas, oranges, blueberries
How many different omelette and fruit combinations are possible?

T. yogurt: vanilla, chocolate, strawberry
toppings: M&M's, hot fudge, pineapple, caramel, granola, Oreo cookie, raspberry
How many different yogurt and topping combinations are possible?

p. 451 (5) Find $P(4,4,4) = \frac{1}{6 \cdot 6 \cdot 6} = \frac{1}{216}$
3 die (6 outcomes)

(6) coin flipped. $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$
$P(\text{all heads or all tails}) = \frac{2}{32} = \frac{1}{16}$

Basic Counting Principle

S. main dishes: hamburger, hot dog, grilled cheese, meat loaf
side dishes: mashed potatoes, cole slaw, French fries, cottage cheese
How many different meals are possible?

M. soups: French onion, clam chowder, chicken noodle, split pea, vegetable
salads: house, Caesar, chef, cobb, pasta
How many different soup and salad combinations are possible?

A. bread: rye, wheat, white, French, sourdough, pumpernickel
fillings: corned beef, ham, chicken salad, roast beef, salami
How many different sandwiches are possible?

E. omelettes: Californian, Western, Spanish, Cheese Lover
fruit: grapefruit, canteloupe, bananas, oranges, blueberries
How many different omelette and fruit combinations are possible?

T. yogurt: vanilla, chocolate, strawberry
toppings: M&M's, hot fudge, pineapple, caramel, granola, Oreo cookie, raspberry
How many different yogurt and topping combinations are possible?

(7) toss a coin 10 times. $= 1024$ outcomes
$2^{10} = 1024$ NOT $2 \cdot 10$

$P(10 \text{ tails}) = \frac{1}{2^{10}} = \frac{1}{1024}$

☆ Factorials

Counting Principle

$0! = 1 \quad 1! = 1 \quad 2! = 2$

Find the value of each factorial.

3·2·1

1. $3!$ 6
2. $6!$ 720 (6·5·4·3·2·1)
3. $4!$ 24
4. $\frac{7!}{5!}$ 42
5. $\frac{8!}{7!}$ = 8
6. $\frac{6!}{2!}$ 360
7. $\frac{3!}{4!}$ $\frac{1}{4}$
8. $\frac{12!}{10!}$ 132

CALCULATOR Find the value of each factorial.

9. $11!$ _____
10. $4! \times 7!$ _____
11. $\frac{13!}{5!}$ _____
12. $\frac{5!8!}{6!}$ _____

MENTAL MATH Find the value of each factorial.

13. $\frac{5!}{3!}$ 20
14. $\frac{18!}{17!}$ 18
15. $\frac{11!}{9!}$ 110
16. $\frac{8!}{6!}$ 56

$\frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1}$ $\quad \frac{18 \cdot 17 \cdot 16 \cdots 1}{17 \cdot 16 \cdots 1}$ $\quad \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdots 1}{9 \cdot 8 \cdot 7 \cdots 1}$

P4

Δ

$\frac{(8 \cdot 7) 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

Probability = $\frac{\text{part}}{\text{whole (total outcomes)}}$

(can be a Fraction, percent or decimal)

deck of cards: 52 cards

26 RED
26 BLACK

4 suites: 13 red hearts, 13 red diamonds
13 black clubs, 13 black spades

Ace High, Number cards 2–10 (9 number cards) ×4 = 36
4 aces
Face cards 3×4 = 12
(Jack, Queen, King)
3 OF each suite

Reduce all Fractions!

Exercises

A Suppose you draw 1 card from a deck of 52 playing cards. Find each probability.

1. *P*(red card) $\frac{26}{52} = \frac{1}{2}$
2. *P*(black card) $\frac{26}{52} = \frac{1}{2}$
3. *P*(ace) $\frac{4}{52} = \frac{1}{13}$
4. *P*(heart) $\frac{13}{52} = \frac{1}{4}$
5. *P*(5) $\frac{4}{52} = \frac{1}{13}$
6. *P*(queen) $\frac{4}{52} = \frac{1}{13}$
7. *P*(king of diamonds) $\frac{1}{52}$
8. *P*(ace of hearts) $\frac{1}{52}$
9. *P*(not a black card) $\frac{26}{52} = \frac{1}{2}$
10. *P*(red 7) $\frac{2}{52} = \frac{1}{26}$
11. *P*(not a heart) $\frac{39}{52} = \frac{3}{4}$
12. *P*(black face card) $\frac{6}{52} = \frac{3}{26}$

red 7 ♥
red 7 ♦

13♢ + 13♡ + 13♧

MULTIPLY

Independent Vs. Dependent Events

① **Independent**: 2 events are independent when the result of the 2nd event does NOT depend on the results of the 1st.

with replacement

① $P(\text{Jack, Jack}) = \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$

② $P(\text{heart, club}) = \frac{13}{52} \cdot \frac{13}{52} = \frac{1}{16}$

② **Dependent**: If the result of the 1st event does affect the results of the 2nd event, and so on. (Without replacement)

* the bottom # in the Fraction will change!

① $P(\text{Jack, Jack}) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$

② $P(\text{heart, club}) = \frac{13}{52} \cdot \frac{13}{51}$

Tell if the events are independent or dependent. Then find each probability.

1. You toss a coin twice. Find P(two heads). Independent (1st Flip, 2nd Flip) $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$
2. Your drawer contains two black socks and two blue socks. Without looking, you take a sock out, hold onto it, then take another sock. Find P(two black socks). Dependent $\frac{2}{4} \cdot \frac{1}{3} = \frac{1}{6}$
3. You toss a coin and roll a number cube marked 1 to 6. Find P(H, odd number). Independent $\frac{1}{2} \cdot \frac{3}{6} = \frac{1}{4}$

A box contains the cards shown at the right.

E	X	P	E	R	I	M	E	N	T

You pick a card without looking. Then you pick a second card without replacing the first. Dependent
Find each probability.

4. P(E, then T) $\frac{1}{30}$ $\frac{3}{10} \cdot \frac{1}{9} = \frac{1}{15}$
5. P(T, then E) $\frac{1}{30}$ $\frac{1}{10} \cdot \frac{3}{9} = \frac{3}{90}$
6. P(M, then X) $\frac{1}{90}$ $\frac{1}{10} \cdot \frac{1}{9} =$
7. P(E, then E) $\frac{1}{15}$ $\frac{3}{10} \cdot \frac{2}{9}$
8. P(E, then not E) $\frac{7}{30}$ $\frac{3}{10} \cdot \frac{7}{9} = \frac{7}{30}$
9. P(not E, then not E) $\frac{7}{10} \cdot \frac{6}{9} = \frac{7}{15}$

Independent Events

Probability WUPS p.7 (1-10)

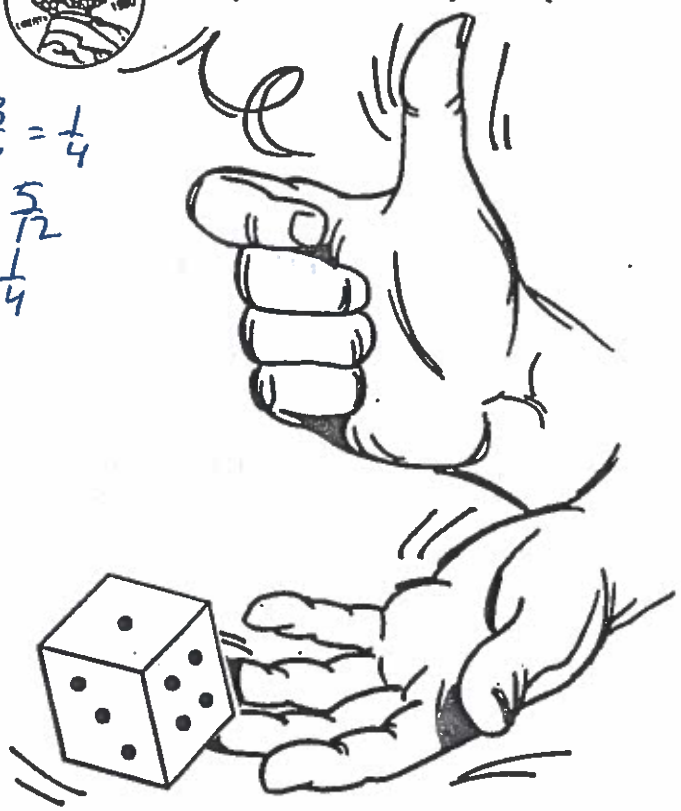
Skill: probability of ordered-pair events

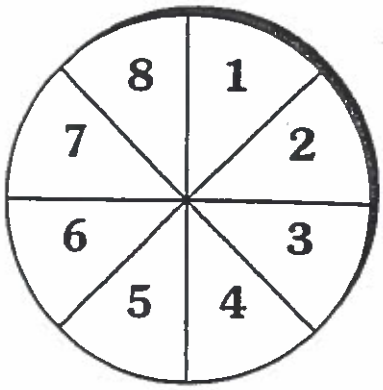
Name _____

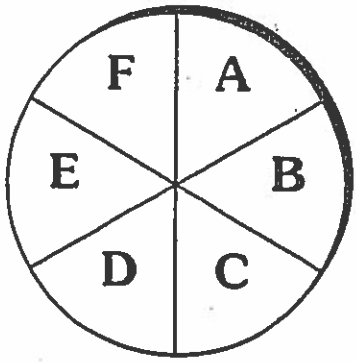
P(event A) • P(event B)

Think of tossing a penny and rolling a die.

1. How many events are possible? $2 \cdot 6 = 12$
2. What is P (H, 4)? $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$
3. What is P (T, an even number)? $\frac{1}{2} \cdot \frac{3}{6} = \frac{1}{4}$
4. What is P (T, a factor of 30)? $\frac{1}{2} \cdot \frac{5}{6} = \frac{5}{12}$
5. What is P (H, number > 3)? $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$
6. What is P (T, an odd number)?
7. What is P (T, not 4)?
8. What is P (H, prime number)?
9. What is P (H, 1 or 6)?
10. What is P (H, factor of 45)?







Think of spinning the two spinners.

11. How many events are possible?
12. What is P (multiple of 2, not A or D)?
13. What is P (prime number, F)?
14. What is P (7, B)?
15. What is P (2 or 3, not C)?
16. What is P (factor of 70, D or E)?
17. What is P (number < 6, A)?
18. What is P (even number, E or C or B)?
19. What is P (factor of 45, C or D)?
20. What is P (1 or 8, not F or E)?

Mutually Exclusive Vs. Not Mutually Exclusive

Events that cannot occur at the same time are mutually exclusive.

$P(\text{Jack or Queen}) = P(\text{Jack}) + P(\text{Queen})$

$P(A \text{ or } B) = P(A) + P(B)$

$\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$

Events can occur at the same time.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

ex $P(\text{Jack or Red}) = P(\text{Jack}) + P(\text{Red}) - P(\text{Red Jack})$

$\frac{4}{52} + \frac{26}{52} - \frac{2}{52}$

$= \frac{28}{52} = \frac{7}{13}$

A single card is drawn from a set of alphabet cards marked A to Z. Tell whether or not the events are mutually exclusive. Find each probability.

1. *P*(K or P) ME $\frac{1}{26} + \frac{1}{26} = \frac{2}{26} = \frac{1}{13}$
2. *P*(a letter before E or a letter after W) M.E. $\frac{4}{26} + \frac{3}{26} = \frac{7}{26}$
3. *P*(a vowel or a letter before M) Not M.E. $\frac{5}{26} + \frac{17}{26} - \frac{3}{26} = \frac{19}{26}$
4. *P*(a consonant or a letter after S) Not M.E. $\frac{11}{13}$ $\frac{21}{26} + \frac{7}{26} - \frac{6}{26} = \frac{22}{26}$
5. *P*(a vowel or a consonant) M.E. $\frac{5}{26} + \frac{21}{26} = \frac{26}{26} = 1$
6. *P*(C or a letter after J) M.E. $\frac{17}{26}$ $\frac{1}{26} + \frac{16}{26} = \frac{17}{26}$

✓

Using a deck of cards, find Each Probability.

① P(red or jack) _____
Not M.E. $P(red) + P(jack) - P(red\ jack)$
$\frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$

② P(black card or red) _____
M.E. $\frac{26}{52} + \frac{26}{52} = \frac{52}{52} = 1$

③ P(jack or queen) _____
M.E. $\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{4}{26} = \frac{2}{13}$
P(J) + P(Q)

④ P(5 or clubs) $\frac{4}{13}$
Not M.E.
$P(5) + P(club) - P(5 \text{ of clubs})$
$\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$

NOT Mutually Exclusive

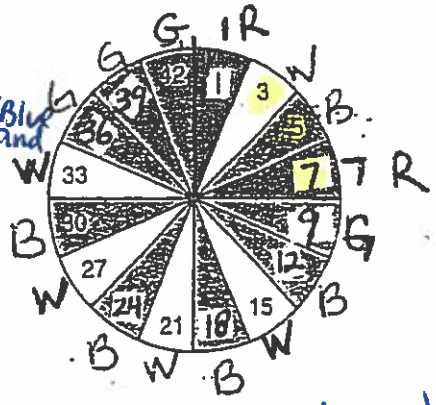
Probability with Overlap

$$P(A \text{ or } B) = P(A) + P(B) - P(A+B)$$

reduce all Fractions

Use the spinner at right. Find each probability.

19. *P*(blue or white)
20. *P*(blue or prime) $P(B) + P(prime) - P(Blue\ and\ prime)$
21. *P*(multiple of 5 or multiple of 4)
22. *P*(odd or blue)
23. *P*(even or green)
24. *P*(green or blue or white)
25. *P*(multiple of 10 or green or red)



19. M.E. $\frac{5}{16} + \frac{5}{16} = \frac{10}{16} = \frac{5}{8}$

20. NOT Mut. Ex. $\frac{5}{16} + \frac{3}{16} - \frac{1}{16} = \frac{7}{16}$

21. ~~NOT~~ ME $\frac{3}{16} + \frac{3}{16} = \frac{6}{16} = \frac{3}{8}$

22.

23. NOT M.E. $P(even) + P(green) - P(even\ +\ green)$
$\frac{6}{16} + \frac{4}{16} - \frac{2}{16} = \frac{8}{16} = \frac{1}{2}$

24.

25.

Permutations
- order matters
- $_nP_r$ (button on calculator)

Formula or $_nP_r = \frac{n!}{(n-r)!}$

r = # of items your picking

n = total # of items

n is always the bigger number

Key words:
- arranged
- places (1st, 2nd, 3rd...)
- letters for words
- numbers
- selected

1. If 5 cars are selected from 8 and arranged side-by-side in a parking lot, how many permutations are possible? $_8P_5 = 6720$
2. How many 3-digit numbers are there using 2, 4, 6, 8 and 9? $_5P_3 = 60$ (5 numbers = n, r)
3. How many arrangements are possible with the numbers 3, 4, 8, 9, 11 and 13? $_6P_6 = 6! = 720$
4. How many ways can 7 horses finish win, place and show? $_7P_3 = 210$
5. How many permutations are there of Annabella, Jake, Sophie, Ellie, Cole, Josie and Miles? $_7P_7 = 7! = 5040$
6. How many 2-digit numbers are there using 1, 2, 3, 4, 5, 6, 7, 8 and 9? $_9P_2 = 72$

7. $_9P_5 =$

8. $_{12}P_4 =$

9. $_4P_4 =$

Combinations

- order does **not** matter
- $_nC_r$ (button on calculator)
- or

Formula $_nC_r = \frac{n!}{(n-r)! \cdot r!}$

total (Bigger)

1. 3 puppies from a litter of 9
2. 5 teachers from a conference of 13
3. 4 piglets from a barn of 8
4. 15 cars from a lot of 18
5. 5 kittens from a litter of 9
6. 4 doughnuts from a dozen

Key words: choose

1. $_9C_3 = 84$
2. $_{13}C_5 = 1287$
3. $_8C_4 = 70$
5. $_9C_5 = 126$
4. $_{18}C_{15} = 816$
6. $_{12}C_4 = 495$

7. $_6C_2 =$

8. $_8C_4 =$

9. $_5C_5 =$

10. $_4C_1 =$

Permutations Name:_____

Find the number of permutations.

1. 1,2 $_2P_2 = 2! = 2$

2. Sue, Kim, Heather 6

3. 12, 14, 16 $_3P_3 = 3! = 6$

4. math, history, English, art $4! = 24$

5. quick, Jane, run $_3P_3 = 6$

6. m, a, t, h $4! = 24$

7. Ace, Queen, Jack 6

8. soup, salad, potatoes, turkey, pie $5! = 120$

Solve.

9. How many 3-letter arrangements can be made from the letters A, B, C, D, E?

$_5P_3 = 60$

10. How many 3-song arrangements can be made from a list of 7 songs?

$_7P_3 = 210$

11. How many 2-letter arrangements are possible the 26 letters? You can use the same letter twice in an arrangement.

$_{26}P_2 = 650$

12. How many 2-digit track jersey numbers can be made from 3, 5, 7, and 9? You can use the same letter twice in an arrangement.

$_4P_2 = 12$

Combinations Name:_____

Tell whether each is a permutation or a combination. Do not solve.

P 1. Ways 5 pictures can be arranged in a row on a wall. $_5P_5 = 5! = 120$

C 2. Ways 3 people can be selected from a group of 5. $_5C_3 = 10$

P 3. Ways first and second violin players can be chosen from 6 players.

C 4. Ways 4 sweaters can be chosen out of 7.

P 5. Ways 5 chairs can be arranged in a row.

P 6. Possible finishing orders of 7 people in a race if there are no ties.

C 7. Ways 3 representatives can be selected from 21 students.

Find the number of combinations.

1. 2 letters from A, B, C

$_3C_2 = 3$

2. 3 people from Blake, Carol, Annie, Phil

$_4C_3 = 4$

3. 4 letters from M, A, T, H

$_4C_4 = 1$

4. 2 symbols from #, $, ?, %, *

$_5C_2 = 10$

5. 3 numbers 10, 20, 30

$_3C_3 = 1$

6. 2 colors from red, orange, green, blue

$_4C_2 = 6$

7. 2 digits from 0 through 5

$_6C_2 = 15$

8. 3 numbers from 1, 2, 3, 4, and 5

$_5C_3 = 10$

9. 1 digit from 0 through 9

$_{10}C_1 = 10$

10. 4 people from Carl, Jim, Sue, Tim

$_4C_4 = 1$

11. 1 shirt from a box of 8 shirts

$_8C_1 = 8$

12. 5 colors from a list of 7

$_7C_5 = 21$

Tell if each exercise is a *permutation* or a *combination*. Then find the answer.

1. the number of ways to line up five trophies on a shelf ${}_5P_5 = 5! = 120$
2. the number of ways to select four books from a group of eight books ${}_8C_4 = 70$
3. the number of ways to elect a three officers from a club of nine people ${}_9C_3 = 84$
4. the number of ways the letters W, I, N, T, E, and R can be arranged ${}_6P_6 = 6! = 720$

Find each answer.

5. ${}_9P_5$ _____ 6. ${}_{10}C_6$ _____ 7. ${}_{12}P_4$ _____ 8. ${}_{12}C_8$ _____

15120

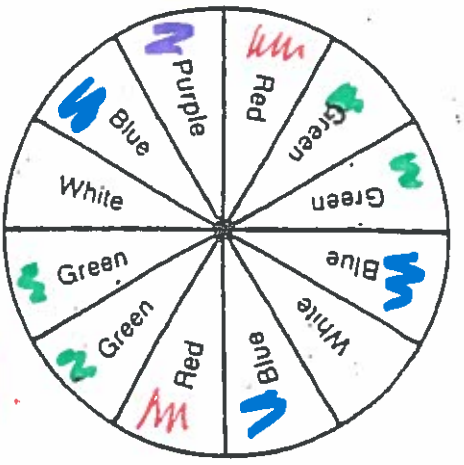
ODDS

odds in favor : $\frac{\#\text{ of favorable outcomes}}{\#\text{ of unfavorable outcomes}}$

odds against : $\frac{\text{against}}{\text{in favor of}}$

You spin the spinner. Find the odds of each event.

13. Odds (red) $\frac{2 \text{ red}}{10 \text{ not red}} = \frac{1}{5}$
14. Odds (blue) $\frac{3}{\text{not blue} \to 9} = \frac{1}{3}$ ← Blue
15. Odds (white) $\frac{2 \text{ white}}{\text{not white } 10} = \frac{1}{5}$
16. Odds (not green) $\frac{8}{\text{not green}} = \frac{8}{4} = 2$
17. Odds (not blue) $\frac{9}{\text{not blue}} = \frac{9}{3} = 3$
18. Odds (yellow) $\frac{0}{\text{not yellow } 12} = 0$
19. Odds (green) _____
20. Odds (purple) _____



$\frac{\text{green}}{\text{not green}} = \frac{4}{8} = \frac{1}{2}$

$\frac{\text{purple}}{\text{not purple}} = \frac{1}{11}$

Probability $P(\text{green}) = \frac{4}{12} = \frac{1}{3}$

$P(\text{purple}) = \frac{1}{12}$ ← total

Name: _____

Practice

Directions: Use the following information to answer Numbers 1 through 4.

Walter wrote the names of the months of the year on identical slips of paper and put them into a hat. He will draw a slip of paper from the hat without looking.

1. What is the probability of Walter drawing a slip of paper with a month that has exactly 4 letters? $\frac{2}{12} = \frac{1}{6}$ June, July

2. What are the odds of Walter drawing a slip of paper with a month that begins with a vowel? $\frac{3 \leftarrow \text{vowel}}{9 \leftarrow \text{not vowel}} = \frac{1}{3}$ April, August, October

3. What is the probability of Walter drawing a slip of paper with a month that has at least 3 letters? 3 or more $\frac{12}{12} = 1$ 100%

at most 3 letters $\frac{1}{12}$

4. What are the odds of Walter drawing a slip of paper with a month that has 31 days? $\frac{4 \leftarrow \text{do have 31}}{8 \leftarrow \text{do not have 31}} = \frac{1}{2}$

Directions: Use the following information to answer Numbers 5 through 8.

The following table shows the number of each color marble in a bag.

Color	Blue	Green	Orange	Red
Number	8	15	5	12

$= 40$ odds(blue) $= \frac{8}{32}$

Kristi will pick a marble out of the bag without looking.

5. What is the probability of Kristi picking a blue marble? $\frac{8}{40} = \frac{2}{10} = \frac{1}{5}$ (part / whole)

6. What are the odds of Kristi picking a green marble? $\frac{15}{25} = \frac{3}{5}$

7. What is the probability of Kristi picking an orange marble? $\frac{5}{40} = \frac{1}{8}$ (not green; total marbles)

8. What are the odds of Kristi picking a red marble? $\frac{12}{28} = \frac{3}{7}$ (not red)

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9. The probability of Joe picking a white golf ball from his bag without looking is $\frac{3}{4}$. What are the odds of Joe picking a white golf ball from his bag?

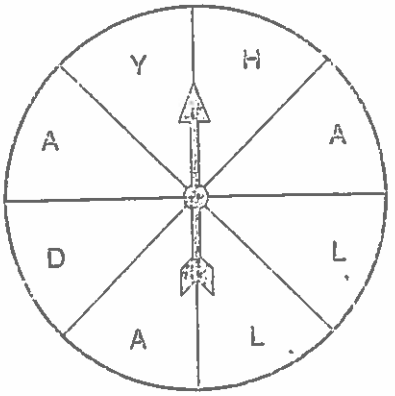
$\frac{3}{1} = 3$

10. The odds of Cheryl picking a red hair tie from her drawer without looking are 2:7. What is the probability of Cheryl picking a red hair tie?

$\frac{2}{9}$

Directions: Use the following information to answer Numbers 11 through 14.

Roy will spin the following spinner once.



11. What is the probability of the spinner landing on the letter L? $\frac{2}{8} = \frac{1}{4}$

12. What are the odds of the spinner landing on the letter A? $\frac{3}{5}$

13. What is the probability of the spinner landing on the letter G? $\frac{0}{8} = 0$

14. What are the odds of the spinner landing on the letter A or the letter Y? $\frac{4}{4} = 1$