

☺ Warm-Ups: ☺

Write in standard form.

- 7. 5^3 125
- 8. $3^{-2} \frac{1}{3^2} \left(\frac{1}{9}\right)$
- 9. 1^9 1
- 11. 18^1 18
- 12. 0.3^2 0.09
- 13. 6^0 1
- 15. $24 \cdot 10^4$ 24,000
- 16. $1.06 \cdot 10^{-3}$ 0.00106
- 17. $8.9 \cdot 10^6$ 8,900,000

Write in scientific notation.

- 19. 467,800 4.678×10^5
- 20. 2,650,000,000 2.65×10^9
- 23. 0.0063 6.3×10^{-3}
- 24. 0.0307 $3.07 \cdot 10^{-2}$

3-8

Laws of Exponents

Goals ■ Apply the laws of exponents.
Applications Astronomy, Sports, Technology

Day 31

1. Find the following products by writing each factor in expanded form. Rewrite the product as a power of ten.

$$\underbrace{10^2}_{10 \cdot 10} \cdot \underbrace{10^3}_{10 \cdot 10 \cdot 10} = 10^5$$

$$10^4 \cdot 10^5 = \underbrace{10^9}_{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} = 10^9$$

$$10^1 \cdot 10^2 = \underbrace{10^3}_{10 \cdot 10 \cdot 10} = 10^3$$

2. How are the exponents in the factors related to the exponent in the product?

Sum of the exponents

3. Find the following quotients by first writing each number in standard form, then dividing. Rewrite the quotient as a power of ten.

$$\underbrace{10^9}_{1,000,000,000} \div \underbrace{10^2}_{100} = 10,000,000 = 10^7$$

$$10^4 \div 10^3 = \underbrace{10^1}_{10} = 10^1$$

$$10^5 \div 10^1 = \underbrace{10^4}_{10,000} = 10^4$$

$$10^{4-3} = 10^1$$

4. How are the exponents in the dividend and divisor related to the exponent in the quotient?

$$10^6 \div 10^2 = 10^{6-2} = 10^4$$

A number can be easier to work with in exponential form. For example, 10^{10} is easier to work with than 10,000,000,000. The laws of exponents help when performing operations with numbers written in exponential form.

To find the product of $2^2 \cdot 2^3$, you can write out the factors for each term and then write the product using exponents.

$$2^2 \cdot 2^3 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$$

The product rule is a faster method.

1 **Product Rule** $a^m \cdot a^n = a^{m+n}$ *Base stays the same*
 To multiply numbers with the same base, write the base with the sum of the exponents. *Add the exponents*

Example 1

Use the product rule to multiply.

a. $10^5 \cdot 10^2$
 10^{5+2}
 $= 10^7$
 10,000,000

b. $5^1 \cdot 5^4$
 5^{1+4}
 5^5

c. $x^2 \cdot x^3$
 x^{2+3}
 x^5

To find the quotient $3^4 \div 3^3$, you could write out the factors for each term and then write the quotient using exponents.

$$3^4 \div 3^3 \text{ can be written as } \frac{3^4}{3^3} = \frac{3 \cdot 3 \cdot 3 \cdot 3}{\cancel{3} \cdot \cancel{3} \cdot 3} = 3^1$$

Another method is to use the quotient rule.

Quotient Rule $a^m \div a^n = a^{m-n}$ *Base stays the same*
 To divide numbers with the same base, write the base with the difference of the exponents. *subtract the exponents!*

Example 2

Use the quotient rule to divide.

a. $10^3 \div 10^1$
 $10^{3-1} = 10^2$
 $10^3 \div 2^2$
 $1000 \div 4$

b. $2^6 \div 2^3$
 2^{6-3}
 2^3

c. $b^7 \div b^6 = b^1$
 $\frac{bbbbbbb}{bbbbbb}$

A number written in exponential form can be raised to a power. You could first write out the factors and then write the product in exponential form.

$$\begin{aligned}
 (4^3)^2 &= 4^3 \cdot 4^3 = 4^6 \\
 &= 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \\
 &= 4^6
 \end{aligned}$$

Compare the exponents in the original number to the exponents in the final number to determine their relationship. This relationship is described by the power rule.

Power Rule	$(a^m)^n = a^{mn}$
	To raise a power to a power, write the base with the product of the exponents.

multiply the exponents
base stays the same!

Example 3

Use the power rule.

a. $(4^3)^6$ *3 · 6 = 18* → 4^{18}

b. $(10^2)^{12}$ *2 · 12 = 24* → 10^{24}

c. $(y^9)^8$ *9 · 8 = 72* → y^{72}

d. $(s)^6$ → s^6

Example 4

ASTRONOMY It is believed that 10^4 craters form on the moon every 10^9 years. On average, how many years are there between the formation of one crater and the next?

Solution

Divide the entire span of 10^9 years by the number of craters, 10^4 .

$$10^9 \div 10^4 = 10^{9-4} = 10^5 = 100,000$$

A crater forms about every 100,000 years.

RETEACHING **3-8**

LAWS OF EXPONENTS

You can use the rules of exponents in simplifying expressions containing numbers in exponential form.

Product Rule: To multiply numbers with the same base, add the exponents:

$$a^m \cdot a^n = a^{m+n}$$

Quotient Rule: To divide numbers with the same base, subtract the exponents:

$$a^m \div a^n = a^{m-n}$$

Power Rule: To raise an exponential number to a power, multiply the exponents:

$$(a^m)^n = a^{m \cdot n}$$

EXERCISES

Use the product rule to simplify.

1. $2^6 \cdot 2^8 = 2^{14}$

2. $5^3 \cdot 5^5 = 5^8$

3. $10^4 \cdot 10^7 = 10^{11}$

4. $4^5 \cdot 4 = 4^6$

Use the quotient rule to simplify. ** rewrite first*

5. $7^8 \div 7 = 7^7$

6. $2^3 \div 2^2 = 2^{3-2} = 2^1 = 2$

7. $4^6 \div 4^6 = 4^0 = 1$

8. $10^9 \div 10^6 = 10^{9-6} = 10^3$

5). $7^8 - 1 = 7^7$
 ~~$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$~~

7). $4^6 - 4 = 4^0 = 1$

Use the power rule to simplify.

9. $(2^3)^4 = 2^{12}$

10. $(5^2)^0 = 5^0 = 1$

11. $(7^2)^{10} = 7^{20}$

12. $(10^3)^6 = 10^{18}$

13. Which of the following are equal to 2^{10} ?

- a. $2^2 \cdot 2^5 = 2^7$
- b. $(2^2)^5 = 2^{10}$
- c. $2^{10} \div 2^0 = 2^{10-0} = 2^{10}$

14. Which of the following are equal to twice 2^{16} ?

- a. 4^{16}
 - b. 2^{17}
 - c. 131,072
 - d. ~~2^{32}~~
- $2 \cdot 2^{16} = 2^{17}$
 131,072

TRY THESE EXERCISES

Use the product rule to multiply.

1. $10^4 \cdot 10^2$

2. $5^8 \cdot 5^5$

3. $m^{21} \cdot m^7$

4. $d^6 \cdot d^3$

Use the quotient rule to divide.

5. $9^8 \div 9^2$

6. $4^{11} \div 4^0$

7. $p^{20} \div p^{10}$

8. $a^9 \div a^3$

Use the power rule.

9. $(3^6)^8$

10. $(7^2)^{10}$

11. $(x^4)^4$

12. $(a^8)^1$

Use the product rule to multiply.

16. $10^{21} \cdot 10^8$

17. $10^1 \cdot 10^9$

18. $5^6 \cdot 5^2$

19. $6^2 \cdot 6^3$

20. $n^3 \cdot n^3$

21. $d^{11} \cdot d^0$

22. $m^9 \cdot m^3$

23. $a^6 \cdot a^9$

Use the quotient rule to divide.

24. $10^4 \div 10^2$

25. $5^8 \div 5^5$

26. $2^8 \div 2^5$

27. $6^{16} \div 6^{15}$

28. $b^9 \div b^1$

29. $x^{21} \div x^7$

30. $y^7 \div y^5$

31. $n^6 \div n^3$

Use the power rule.

32. $(45^6)^{10}$

33. $(9^8)^8$

34. $(5^7)^9$

35. $(11^4)^5$

36. $(a^2)^{15}$

37. $(x^{20})^5$

38. $(n^7)^7$

39. $(p^3)^3$

Use the laws of exponents.

40. $(4^{10})^{15}$

41. $3^{16} \div 3^8$

42. $8^{24} \div 8^3$

43. $15^{18} \cdot 15^{14}$

44. $7^{15} \div 7^3$

45. $6^{20} \cdot 6^5$

46. $(12^5)^5$

47. $2^{90} \cdot 2^9$

Find the value of each variable.

48. $6^3 \cdot 6^a = 6^{18}$

49. $7^4 \div 7^b = 7^2$

50. $(6^c)^3 = 6^{12}$

51. $(9^7)^f = 9^{49}$