RETEACHING 3-1

ADD AND SUBTRACT SIGNED NUMBERS

To add integers on a number line:

- start with zero;
- move to the right for a positive integer;
- move to the left for a negative integer.

Add: Same Signs? Add numbers + Keep Sign
different Signs? Subtract numbers + Keep Sign of larger number

Subtract: Add the Opposite + then follow Add rules

EXERCISES

Draw a number line and add.

1. $-4 + (-4)$ _____
2. $12 + (-8)$ _____
3. $-5 + (-8)$ _____
4. $-2 + (-9)$ _____
5. $6 + (-3)$ _____
6. $-3 + (-7)$ _____

Subtract.

7. $3 - 7$ _____
8. $8 - 7 - (-6)$ _____
9. $9 - 5 - 17$ _____
10. $15 - (-8)$ _____
11. $-21 - (-8)$ _____
12. $19 - (-7)$ _____
13. $-14 - 6$ _____
14. $-3 - (-32)$ _____
15. $-6 - 6$ _____
16. $-9 - (-9)$ _____
17. $34 - 43$ _____
18. $16 - (-1)$ _____

RETEACHING 3-2

MULTIPLY AND DIVIDE SIGNED NUMBERS

The rules for multiplying integers are as follows:

- The product of two integers having the same signs is positive.
- The product of two integers having opposite signs is negative.

The rules for dividing integers are as follows:

- The quotient of two integers having the same signs is positive.
- The quotient of two integers having opposite signs is negative.

EXERCISES

Use the rules for multiplying integers to find each product.

1. $-8 \cdot 5$ _____ **2.** $7 \cdot (-11)$ _____ **3.** $-9 \cdot (-7)$ _____

4. $3 \cdot 12$ _____ **5.** $-8 \cdot (-10)$ _____ **6.** $-4 \cdot 9$ _____

7. $-6 \cdot (-6)$ _____ **8.** $8 \cdot (-6)$ _____ **9.** $-2 \cdot (-30)$ _____

10. $3 \cdot 5 \cdot (-9)$ _____ **11.** $-6 \cdot 5 \cdot 8$ _____

12. $-5 \cdot (-6) \cdot (-5)$ _____ **13.** $-4 \cdot (-7) \cdot 2$ _____

Find each quotient. Then check by multiplying.

14. $14 \div (-7)$ _____ **15.** $-32 \div (-4)$ _____ **16.** $-63 \div 7$ _____

17. $-54 \div (-9)$ _____ **18.** $-72 \div 8$ _____ **19.** $-18 \div (-3)$ _____

RETEACHING 3-3

ORDER OF OPERATIONS

It would be very confusing if people got different answers when evaluating an expression. To avoid that problem, operations are always done in a specified order.

1. Parentheses—perform all calculations within grouping symbols first.
2. Exponents—do all calculations with exponents.
3. Multiplication and division—multiply and divide in order from left to right.
4. Addition and subtraction—add or subtract in order from left to right.

EXERCISES

What operation should you do first?

1. $7 \cdot 3 - 5 + 2$ _____

2. $74 - (2 \cdot 3)^2 + 1$ _____

3. $56 \div (4 + 4) \cdot 3$ _____

4. $2^3 + 5 \cdot 3$ _____

5. $210 - 7 \cdot 8 \div 4$ _____

6. $25 - 16 + 2 - 8$ _____

11. $2 \cdot 2 + 0 \cdot (18 \div 56)$ _____

12. $(25 - 8) + 6^2 - 15$ _____

RETEACHING 3-4

REAL NUMBER PROPERTIES

Commutative Property	The order of addends or factors does not affect the answer.	$a + b = b + a$ $a \cdot b = b \cdot a$	$3 + 2 = 5; 2 + 3 = 5$ $7 \cdot 4 = 28; 4 \cdot 7 = 28$
Associative Property	The grouping of addends or factors does not affect the answer.	$a + (b + c) = (a + b) + c$ $a \cdot (b \cdot c) = (a \cdot b) \cdot c$	$3 + (4 + 5) = (3 + 4) + 5$ $4 \cdot (2 \cdot 3) = (4 \cdot 2) \cdot 3$
Distributive Property	A factor outside the parentheses can be used to multiply each term within the parentheses.	$a(b + c) = (a \cdot b) + (a \cdot c)$ $a(b - c) = (a \cdot b) - (a \cdot c)$	$6(2 + 3) = (6 \cdot 2) + (6 \cdot 3)$ $3(12 - 7) = (3 \cdot 12) - (3 \cdot 7)$

EXERCISES

Match each equation with the property illustrated.

1. $5 \cdot (2 \cdot 2) = 10 \cdot 2$ _____

2. $2 \cdot 7 = 7 \cdot 2$ _____

3. $4\,(2 + 7) = (4 \cdot 2) + (4 \cdot 7)$ _____

4. $15 + (21 + 7) = 15 + (7 + 21)$ _____

5. $(8 + 3) + 17 = 8 + (3 + 17)$ _____

a. commutative property of addition

b. commutative property of multiplication

c. associative property of addition

d. associative property of multiplication

e. distributive property

Complete.

6. _____ $\cdot 73 = 73 \cdot 10$

7. $(5.6 + 8.2) +$ _____ $= 5.6 + (8.2 + 1.8)$

8. $837 +$ _____ $= 16 + 837$

9. $6 \cdot (5 \cdot 12) = (6 \cdot$ _____$) \cdot 12$

10. $2(3 + 8) = ($_____$\cdot 3) + (2 \cdot 8)$

11. _____$(5 - 2) = (3 \cdot 5) - (3 \cdot 2)$

12. $4(18 - 2) = (4 \cdot$ _____$) - (4 \cdot 2)$

13. $0.3(4 + 9) = (0.3 \cdot 4) + (0.3 \cdot$ _____$)$

RETEACHING **3-5**

VARIABLES AND EXPRESSIONS

A **variable** is a letter that is used to represent an unknown number. To **evaluate** an expression means to find the value of the expression for a particular value of the variable or variables.

Variables
a, b

Variable Expressions
$3x + 2$
variables
$n - 5$

EXERCISES

Match each variable expression with its meaning.

1. $\frac{n}{2}$ _____
2. $2 + n$ _____
3. $2n$ _____
4. $n - 2$ _____
5. $\frac{2}{n}$ _____

a. 2 more than a number
b. 2 divided by a number
c. 2 less than a number
d. twice a number
e. a number divided by 2

Write an expression for each situation.

6. 12 cookies divided among n people _____
7. 8 more than n dollars _____
8. 7 less than d compact discs _____
9. 8 times x people _____

Evaluate each expression. Let $n = 18$.

10. $n - 5$ _____ **11.** $8n$ _____ **12.** $n \div 9$ _____ **13.** $14 + n$ _____

Complete each table.

14.

n	$6n$
0	
1	
2	
3	

15.

a	$12 - a$
12	
11	
10	
6	

16.

t	$t \div 6$
0	
6	
12	
180	

Name _____ Date _____

ENRICHMENT 3-5

VARIABLE MAGIC

You can use what you know about writing expressions with variables to analyze number tricks. Examine the following table, which gives the steps in one number trick. The variable expressions help you understand how the trick works.

Number Operations	Example	Your Number	Variable Expression
1. Pick any whole number.	3		x
2. Add 7.	10		$x + 7$
3. Now double the result.	20		$2x + 14$
4. Subtract 4.	16		$2x + 14 - 4$
5. Take one half of the result.	8		$\frac{2x}{2} + \frac{10}{2}$ or $x + 5$
6. Subtract your original number.	5		$x + 5 - x$ or 5

EXERCISES

1. Complete the table in the example above using your own number.

2. Write variable expressions to complete the table.

Number Operations	Variable Expression
1. Pick any whole number.	
2. Double the number.	
3. Multiply by 4.	
4. Add 40.	
5. Divide by 8.	
6. Subtract your original number.	

3. Write variable expressions to complete the table.

Number Operations	Variable Expression
1. Pick any whole number.	
2. Triple the number.	
3. Add 12.	
4. Double the result.	
5. Divide by 6.	
6. Subtract your original number.	

RETEACHING 3-6

PROBLEM SOLVING SKILLS: FIND A PATTERN

In solving some problems, sometimes you can organize the information in a table that will show a pattern.

Example

Every day, Maria saves twice as many pennies as she did the day before. If Maria begins the year by putting away one penny, how many pennies will she have by January 10?

Solution

Make a table.

Date	1	2	3	4	5	6	7	8	9	10
Number	1	2	4	8	16	32	64	128	256	512

Add the total number of pennies saved.

$1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 = 1023$

Maria will have saved 1023 pennies.

EXERCISES

Look for a pattern and choose the rule from those given. Then write the unknown numbers.

1. 0.24, 0.48, 0.72, ■, ■ _____
   - **a.** Double the previous number.
   - **b.** Add 0.24 to the previous number.
   - **c.** Add 0.024 to the previous number.

2. 12.6, 12.4, ■, 12.0, ■ _____
   - **a.** Add 0.2 to the previous number.
   - **b.** Subtract 0.2 from the previous number.
   - **c.** Subtract 0.02 from the previous number.

3. Find each product and look for a pattern.

   $10 \cdot 10 =$ _____ $15 \cdot 15 =$ _____

   $9 \cdot 11 =$ _____ $14 \cdot 16 =$ _____

4. If $45 \cdot 45$ is 2025, what is $44 \cdot 46$? _____

5. If $33 \cdot 35$ is 1155, what is $34 \cdot 34$? _____

6. Nikia put $10 in her savings account in January, $13 in February, $16 in March, and so on. If the pattern continues, how much money will she put into her account in December? How much will she have put into the account for the entire year?

Name _____ Date _____

RETEACHING 3-7

EXPONENTS AND SCIENTIFIC NOTATION

Multiples of a number can be written in **exponential form**. The base tells what factor is being multiplied. The exponent tells how many equal factors there are.

$6 \cdot 6 \cdot 6 = 6^3$ (exponent; base) Read "six to the third power."

Very large and very small numbers can be written as the product of a number greater than or equal to 1 but less than 10 and a power of 10. A number expressed in this form is in **scientific notation**.

Example 1

a. Write $3 \cdot 3 \cdot 3 \cdot 3$ in exponential form. **b.** Write 7^5 in standard form.

Solution

a. $3 \cdot 3 \cdot 3 \cdot 3 = 3^4$ 4 factors

b. $7^5 = 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 16{,}807$ 5 factors

Example 2

a. Write 5,600,000 in scientific notation.
b. Write $3.8 \cdot 10^2$ in scientific form.

Solution

a. Move the decimal point to the right to get a number greater than or equal to 1 and less than 10.

5,600,000. ← 6 places

Write 5,600,000 as the product of 5.6 and a power of 10 equal to the number of decimal places you moved the decimal point.

$5{,}600{,}000 = 5.6 \cdot 1{,}000{,}000 = 5.6 \cdot 10^6$

b. $3.8 \cdot 10^2 = 3.8\ \ 100 = 380$

EXERCISES

Write in exponential form.

1. $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ _____ **2.** $10 \cdot 10 \cdot 10 \cdot 10$ _____

Write in standard form.

3. 7^3 _____ **4.** 8^0 _____ **5.** 2^6 _____ **6.** 30^2 _____

Write in scientific notation.

7. 3700 _____ **8.** 24,000,000 _____

Write in standard form.

9. $1.6 \cdot 10^4$ _____ **10.** $3.088 \cdot 10^8$ _____

RETEACHING 3-8

LAWS OF EXPONENTS

You can use the rules of exponents in simplifying expressions containing numbers in exponential form.

Product Rule: To multiply numbers with the same base, add the exponents:

$a^m \cdot a^n =$ a^{m+n}

Quotient Rule: To divide numbers with the same base, subtract the exponents:

$a^m \div a^n =$ a^{m-n}

Power Rule: To raise an exponential number to a power, multiply the exponents:

$(a^m)^n = a^{m \cdot n}$

EXERCISES

Use the product rule to simplify.

1. $2^6 \cdot 2^8$ _____ **2.** $5^3 \cdot 5^6$ _____

3. $10^4 \cdot 10^7$ _____ **4.** $4^5 \cdot 4$ _____

Use the quotient rule to simplify. ✱ rewrite first

5. $7^8 \div 7$ _____ **6.** $2^3 \div 2^2$ _____ **7.** $4^6 \div 4^6$ _____ **8.** $10^9 \div 10^6$ _____

5). _____ 6). _____ 7). _____ 8) _____

Use the power rule to simplify.

9. $(2^3)^4$ _____ **10.** $(5^2)^0$ _____ **11.** $(7^2)^{10}$ _____ **12.** $(10^3)^6$ _____

13. Which of the following are equal to 2^{10}?
a. $2^2 \cdot 2^5$ **b.** $(2^2)^5$ **c.** $2^{10} \div 2^0$

14. Which of the following are equal to twice 2^{16}?
a. 4^{16} **b.** 2^{17} **c.** 131,072 **d.** 2^{32}

RETEACHING 3-9

SQUARES AND SQUARE ROOTS

The square of 6 is 36, and the square of -6 is 36.

$$6 \cdot 6 = 6^2 = 36 \text{ and } (-6) \cdot (-6) = (-6)^2 = 36$$

6 is the *positive square root* of 36. $\sqrt{36} = 6$

-6 is the *negative square root* of 36. $-\sqrt{36} = -6$

Any number whose square roots are integers is a **perfect square**. For example, the numbers 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100 are perfect squares. The square root of all numbers that are not perfect squares are **irrational numbers**—nonterminating, nonrepeating decimals. The square roots of these numbers are given as approximations, usually rounded to the nearest thousandth. The square root of 2, for example, is 1.414.

EXERCISES

Find each square.

1. 19^2 **2.** 22^2 **3.** 25^2 **4.** $(-16)^2$ **5.** 39^2 **6.** $(-28)^2$

7. $(-0.19)^2$ **8.** $(1.6)^2$ **9.** $(3.2)^2$ **10.** $\left(\frac{5}{7}\right)^2$ **11.** $\left(\frac{8}{9}\right)^2$ **12.** $\left(\frac{3}{4}\right)^2$

Find each square root.

13. $\sqrt{1024}$ **14.** $\sqrt{676}$ **15.** $-\sqrt{289}$ **16.** $\sqrt{0.81}$ **17.** $\sqrt{0.0049}$ **18.** $-\sqrt{2.25}$

19. $\sqrt{27}$ **20.** $-\sqrt{19}$ **21.** $\sqrt{89}$ **22.** $-\sqrt{51}$ **23.** $\sqrt{75}$ **24.** $-\sqrt{13}$