Day 37

4.4 Geometric Transformations with Matrices

Preimage - original figure
Before

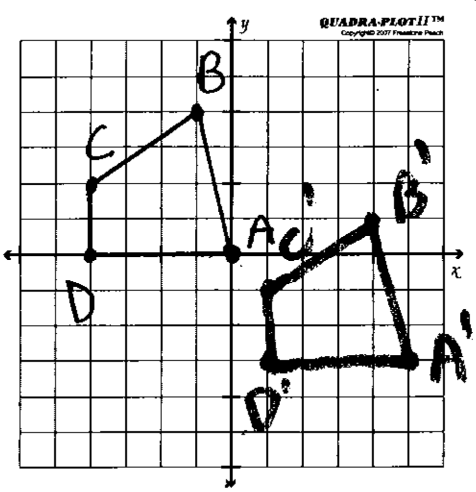
Image - figure after the transformation
After

① Translation (Slide) - moves a figure up/down and left/right
- doesn't change size

ex Translate quadrilateral ABCD 5 units right and 3 down

A(0,0) B(-1,4) C(-4,2) D(-4,0)

A'(5,-3) B'(4,1) C'(1,-1) D'(1,-3)



$$\begin{array}{c} \\ x \\ y \end{array}\begin{array}{c}\begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & -1 & -4 & -4 \\ 0 & 4 & 2 & 0 \end{bmatrix}\end{array} + \underbrace{\begin{bmatrix} 5 & 5 & 5 & 5 \\ -3 & -3 & -3 & -3 \end{bmatrix}}_{\text{translation matrix}}$$

$$\boxed{= \begin{array}{c}\begin{bmatrix} 5 & 4 & 1 & 1 \\ -3 & 1 & -1 & -3 \end{bmatrix} \\ \begin{array}{cccc} A' & B' & C' & D' \end{array}\end{array}}$$

ex What matrix would you use to translate a pentagon 3 units left and 2 units up?

$$\begin{array}{c} x \\ y \end{array}\begin{bmatrix} -3 & -3 & -3 & -3 & -3 \\ 2 & 2 & 2 & 2 & 2 \end{bmatrix} \qquad (2 \times 5)$$

right - positive x
left - negative x

up = positive y
down - negative y

(2) Dilation - enlarge or reduce the size of a figure, same shape, different size
(similar figures)

If the center of dilation is at the origin (0,0), you can use scalar multiplication to find the vertices of the image.

wkst 1. dilation of 11

A (4, 2)
B (4, -2)
C (-3, 0)

$$11\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 4 & 4 & -3 \\ 2 & -2 & 0 \end{bmatrix} \end{array}$$

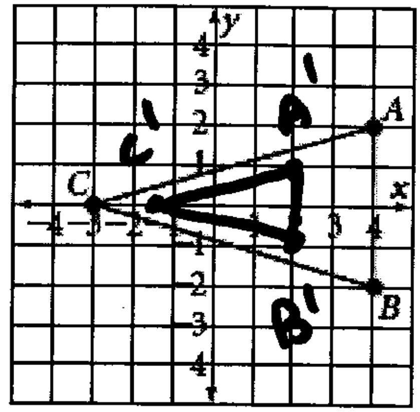
$$= \begin{bmatrix} 44 & 44 & -33 \\ 22 & -22 & 0 \end{bmatrix}$$

Name ______________________ Class ________________ Date ____________

Practice 4-4 .... Geometric Transformations With Matrices

For Exercises 1–11, use ΔABC at the right. Find the coordinates of the image under each transformation. Express your answer as a matrix.

1. a dilation of 11



2. a translation 1 unit right and 4 units up

$$\begin{bmatrix} 4 & 4 & -3 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 4 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 5 & -2 \\ 6 & 2 & 4 \end{bmatrix}$$

3. a dilation of 1.5

$$1.5 \begin{bmatrix} 4 & 4 & -3 \\ 2 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 6 & -4.5 \\ 3 & -3 & 0 \end{bmatrix}$$

4. a translation 2 units right and 6 units down

5. a reflection in $y = x$

6. a rotation of 270°

7. a rotation of 90°

8. a translation 1 unit left and 2 units down

9. a translation 3 units left and 1 unit up

10. a dilation of $\frac{1}{2}$

$$\frac{1}{2} \begin{bmatrix} 4 & 4 & -3 \\ 2 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 & -1.5 \\ 1 & -1 & 0 \end{bmatrix}$$

A' (2,1)

B' (2,-1)

C' (-1.5,0)

11. a reflection in the x-axis

③ Use matrix multiplication to graph Reflections in the coordinate plane. FLIP

y-axis	x-axis	y = x	y = -x
$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

HW: p.195 (1–9, 27–30)

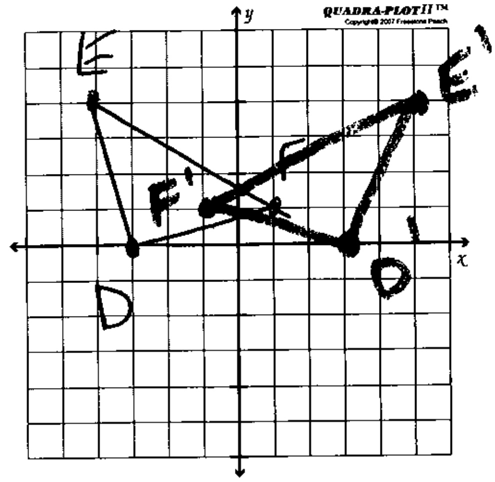
Graphs

Day 38

ex Reflect D(−3,0) E(−4,4) F(1,1)

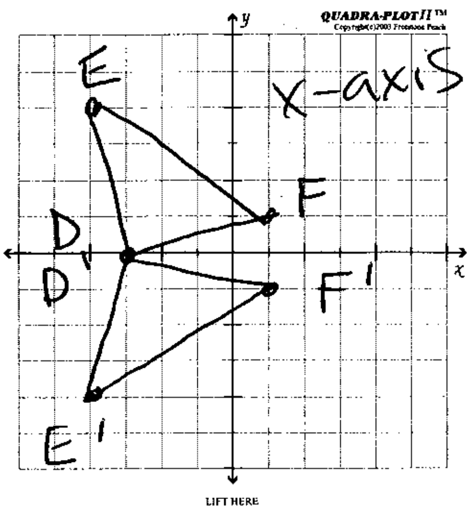
over the y-axis

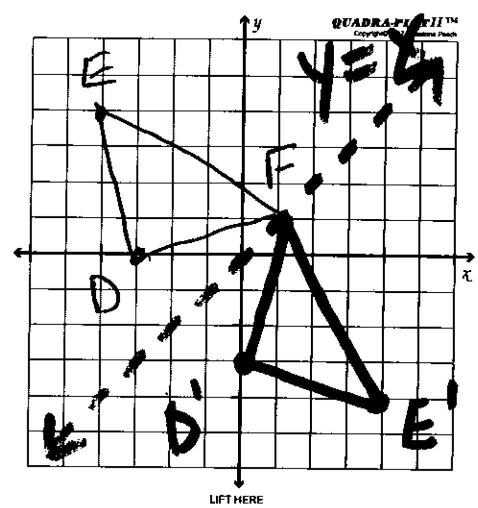
$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{array}{c} \begin{bmatrix} -3 & -4 & 1 \\ 0 & 4 & 1 \end{bmatrix} \\ \begin{array}{ccc} D & E & F \end{array} \end{array} = \begin{bmatrix} 3 & 4 & -1 \\ 0 & 4 & 1 \end{bmatrix}$$



(b) x-axis

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -3 & -4 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -4 & 1 \\ 0 & -4 & -1 \end{bmatrix}$$





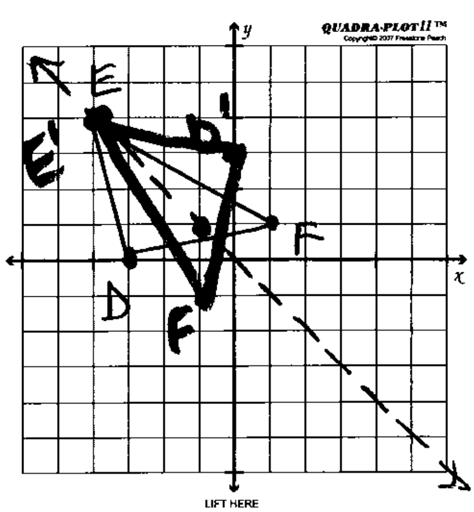
(c) $y = x$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -3 & -4 & 1 \\ 0 & 4 & 1 \end{bmatrix}$

$m = \frac{1}{1}$

$b = 0$ $= \begin{bmatrix} 0 & 4 & 1 \\ -3 & -4 & 1 \end{bmatrix}$

D'

(d) $y = -x$ $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -3 & -4 & 1 \\ 0 & 4 & 1 \end{bmatrix}$



$= \begin{bmatrix} 0 & -4 & -1 \\ 3 & 4 & -1 \end{bmatrix}$

D' E' F

Rotations — counterclockwise about the origin G

$90^\circ \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ $180^\circ \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$270^\circ \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ $360^\circ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Rotate A(1,1) B(3,1) C(6,4) D(1,3)

ex 90° $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 3 & 6 & 1 \\ 1 & 1 & 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} -1 & -1 & -4 & -3 \\ 1 & 3 & 6 & 1 \end{bmatrix}$

$\quad$ A' B' C' D'