

## 8.4 Properties of Logarithms

①  $\log_b M \cdot N = \log_b M + \log_b N$

Product Property

expand  $\rightarrow$

condense  $\leftarrow$  (write as a single logarithm)

ex) Expand  $\log_2 7 \cdot X = \log_2 7 + \log_2 X$

### ② Quotient Property

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

ex) Expand  $\log_2 \frac{X}{2} = \log_2 X - \log_2 2$

$$\log_2 X - 1$$

### ③ Power Property

$$\log_b M^x = x \log_b M$$

expand  $\rightarrow$

\* move exponent in front when expanding.

ex) Expand  $\log_7 a^3 \cdot b^4$

$$\log_7 a^3 + \log_7 b^4$$

$$3 \log_7 a + 4 \log_7 b$$

Expand.

$$\begin{aligned}\textcircled{1} \quad \log\left(\frac{y}{3}\right)^2 &= \log\left(\frac{y^2}{9}\right) \\ &= \log y^2 - \log 9 \\ &\quad \uparrow \qquad \qquad \qquad \uparrow \\ &\quad \text{numerator} \qquad \text{denominator} \\ &= \boxed{2 \log y - \log 9}\end{aligned}$$

$$\begin{aligned}\textcircled{2} \quad \log_4 5\sqrt{x} &= \log_4 5x^{\frac{1}{2}} \\ &= \log_4 5 + \log_4 x^{\frac{1}{2}} \\ &= \boxed{\log_4 5 + \frac{1}{2} \log_4 x}\end{aligned}$$

$$\begin{aligned}\textcircled{3} \quad \log \frac{3m^4}{n^2} &= \log 3m^4 - \log n^2 \\ &= \log 3 + \log m^4 - \log n^2 \\ &= \log 3 + 4 \log m - 2 \log n\end{aligned}$$

Condense. (write as a single logarithm)

1st:  
move  
#s  
in  
front to  
exponents

$$\begin{aligned}\textcircled{1} \quad 3 \log 2 + \log 4 - \log 16 \\ \log 2^3 + \log 4 - \log 16 \\ \log 8 + \log 4 - \log 16 \\ \log 32 - \log 16 = \log \frac{32}{16} = \log 2\end{aligned}$$

$$\textcircled{2} \quad \log_4 64 - \log_4 16 = \log_4 \frac{64}{16} = \log_4 4$$

$$\log_4 4 = x$$

$$4^x = 4^1$$

$$x = 1$$

$$= \textcircled{1}$$

$$\textcircled{3} \quad 6 \log_5 x + \log_5 y$$

$$\log_5 x^6 + \log_5 y$$

$$\log_5 x^6 y$$

$$\textcircled{4} \quad 2 \log w - 4 \log 3$$

$$\log w^2 - \log 3^4$$

$$\log w^2 - \log 81$$

$$\log \left( \frac{w^2}{81} \right)$$

Use the properties of logarithms to evaluate the expression. **1st: condense**  
**2nd: evaluate.**

$$\textcircled{1} \quad \log_2 4 - \log_2 16 = -2$$

$$\log_2 \frac{4}{16} = \log_2 \frac{1}{4} = x$$

$$2^x = \frac{1}{4} = \frac{1}{2^2}$$

$$2^x = \frac{1}{2^2}$$

$$2^x = 2^{-2}$$

$$x = -2$$

$$\textcircled{2} \log_3 3 + 5 \log_3 3$$

$$\begin{array}{r} 1 + 5 \cdot 1 \\ 1 + 5 \end{array}$$

$\textcircled{6}$

$$\textcircled{3} \log_6 4 + \log_6 9 = 2$$

$$\log_6 36 \leftarrow (4 \cdot 9)$$

$$\log_6 36 = X$$

$$6^x = 36$$

$$6^x = 6^2$$

$$\textcircled{x=2}$$

CW:

p. 457

(12 - 20 Even,

23, 24, 26,

34 - 40 Even)