Day 64

# 6.3 Synthetic Division and the Remainder Theorem

**Synthetic Division** only works for divisors that are linear binomials with a leading coefficient of 1.

(ex) $x+1$, $m-3$, $y-7$

NOT $2x+5$ (linear but 2 is the leading coefficient), $y^2-1$ (quadratic binomial), $x^2+x-3$ (quadratic trinomial)

(ex) $(2x^4+6x^3+5x^2-45) \div (x+3)$

$x+3=0$, $-3\ -3$, $x=-3$

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| | $x^4$ | $x^3$ | $x^2$ | $x$ | constant |
|---|---|---|---|---|---|
| $-3$ | 2 | 6 | 5 | 0 | −45 |
| | ↓ | −6 | 0 | −15 | 45 |
| | 2 | 0 | 5 | −15 | 0 ← remainder |
| | $x^3$ | $x^2$ | $x$ | constant | |

$$2x^3+5x-15$$

(ex) $(x^4-5x^2+4x+12) \div (x+2)$

| | $x^4$ | $x^3$ | $x^2$ | $x$ | constant |
|---|---|---|---|---|---|
| $-2$ | 1 | 0 | −5 | 4 | 12 |
| | | −2 | 4 | 2 | −12 |
| | 1 | −2 | −1 | 6 | 0 |

$$x^3-2x^2-1x+6$$

$(x+2)(x^3-2x^2-1x+6) = x^4-5x^2+4x-12$

The Remainder theorem says you can evaluate a polynomial using synthetic ÷, and the remainder is your answer.

(ex) Find $P(-1)$ for $P(x) = 2x^4 + 6x^3 - 5x^2 - 60$

**Old way:** $P(x) = 2(-1)^4 + 6(-1)^3 - 5(-1)^2 - 60$

**2nd old way:** $y = 2x^4 + 6x^3 - 5x^2 - 60$

go to table Find y when $x = -1$

3rd: $P(-1)$ that means $x = -1$

| | $x^4$ | $x^3$ | $x^2$ | $x$ | constant |
|---|---|---|---|---|---|
| $-1$ | 2 | 6 | $-5$ | 0 | $-60$ |
| | ↓ | $-2$ | $-4$ | 9 | $-9$ |
| | 2 | 4 | $-9$ | 9 | $-69$ |

$P(-1) = -69$

(ex) Find $P(-2)$ for

$P(x) = x^3 + 7x^2 + 4x$

| | | | | |
|---|---|---|---|---|
| $-2$ | 1 | 7 | 4 | 0 |
| | | $-2$ | $-10$ | 12 |
| | 1 | 5 | $-6$ | 12 |

this graph has a point on it at $(-2, 12)$

$P(-2) = 12$

(ex) $y = x^3 + 2x^2 - 5x - 6$ ÷ by $(x+1)$

use synthetic ÷ to Factor completely.

$x+1=0$
$-1\ -1$
$x=-1$

$$\begin{array}{r|cccc} -1 & 1 & 2 & -5 & -6 \\ & \downarrow & -1 & -1 & 6 \\ \hline & 1x^2 & 1x & -6 & 0 \end{array}$$

Factor all the way.

$x^2+x-6$

$(x-2)(x+3)$

$-6$

$-1 \mid +6 = -1$

$-2 \mid 3$

$2 \mid -3$

$1 \mid -6$

$x^3+2x^2-5x-6$

Factors into

$(x+1)(x-2)(x+3)$

divisor answer

p. 324-325

(2-32 even, 40, 41, 55)

L.D. 2,4,6,8,40,41

Name _____ Period ____

standard Form $2x^3-7x^2-8x+16$

coefficients from the dividend

# Synthetic Division (cont.)

Use synthetic division to divide the polynomials.

4. $(16 - 8x - 7x^2 + 2x^3) \div (x - 4)$

1st: $x-4=0$, $+4\ +4$, $x=4$

$$\begin{array}{c|cccc} 4 & 2 & -7 & -8 & 16 \\ & \downarrow & 8 & 4 & -16 \\ \hline & 2x^2 & 1x & -4 & \boxed{0} \leftarrow \text{remainder} \end{array}$$

$2x^2$: quadratic term; $1x$: linear term; $-4$: constant

answer = $2x^2+1x-4$

5. ~~$(2x^2 + 3 + 5x) \div (x + 1)$~~

$(x-4)(2x^2+1x-4)=$ ← factors into

divisor · answer

$2x^3-7x^2-8x+16$

dividend

6. $(3x + x^2 - 18) \div (x + 3)$

$(1x^2+3x-18) \div (x+3)$

$x+3=0$

$-3\ -3$

$x=-3$

$$\begin{array}{c|ccc} -3 & 1 & 3 & -18 \\ & \downarrow & \text{add } -3 & 0 \\ \hline & 1 & 0 & \boxed{-18} \leftarrow \text{remainder} \end{array}$$

multiply

1: linear; 0: constant

$1x+0+\frac{-18}{x+3}$

$1x+\frac{-18}{x+3}$

7. $(2x^2 - 4x + 3) \div (x - 3)$

8. $(x - x^2 + 8 + x^3) \div (x - 1)$

9. $(-x^2 + 2x - 4 + 3x^3) \div (x - 2)$

Name_____ Period____

# Dividing Polynomials

Divide each polynomial expression. Check your answer.

**Example:** $\frac{(4x^2 - 2x + 6)}{(2x - 3)}$

$$\begin{array}{r} 2x + 2 \\ 2x - 3 \overline{)\,4x^2 - 2x + 6} \\ \underline{-4x^2 + 6x} \\ 4x + 6 \\ \underline{-4x + 6} \\ 12 \text{ remainder} \end{array}$$

$$(2x + 2) + \frac{12}{2x - 3}$$

**Check:** $(2x + 2)(2x - 3) + 12$

$4x^2 - 2x - 6 + 12 = 4x^2 - 2x + 6$

1. $(x^3 - 1) \div (x^2 - 1)$

2. $(4x^4 + 5) \div (x^2 + 1)$

3. $\frac{(x^2 - 3x - 7)}{x + 2}$

4. $(x^3 - 6) \div (x - 1)$

5. $(x^3 - 6x^2 + 1) \div (x + 2)$

6. $\frac{2x^4 + 2x^3 - 10x - 9}{x^3 + x^2 - 5}$

7. $\frac{10x^3 + 27x^2 + 14x + 5}{x^2 + 2x}$

8. $\frac{(6x^2 - x - 7)}{(3x + 1)}$