3-5

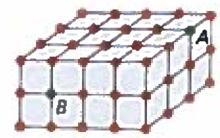
Multiple Representations

Graphs in Three Dimensions

Day 30

1 Graphing Points in Three Dimensions

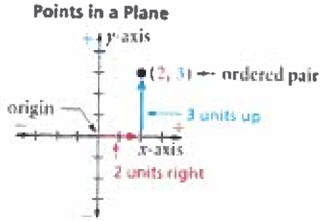
Suppose you want to describe how to get from point *A* to point *B* along the grid shown at the right. You could say "Move down one unit, forward two units, and left three units," or "Move left three units, forward two units, and down one unit."



To describe positions in space, you need a three-dimensional coordinate system.

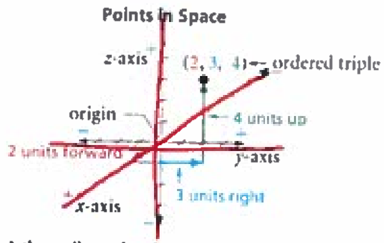
You have learned to graph on an *xy*-coordinate plane using ordered pairs. Adding a third axis, the *z*-axis, to the *xy*-coordinate plane creates **coordinate space.** In coordinate space you graph points using **ordered triples** of the form (x, y, z).

Points in a Plane



A two-dimensional coordinate system allows you to graph points in a plane.

Points in Space



A three-dimensional coordinate system allows you to graph points in space.

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In the coordinate plane, point (2, 3) is two units right and three units up from the origin. In coordinate space, point (2, 3, 4) is two units forward, three units right, and four units up.

Points in space have an ordered triple.

$$(x, y, z)$$

up or down

Forward backwards

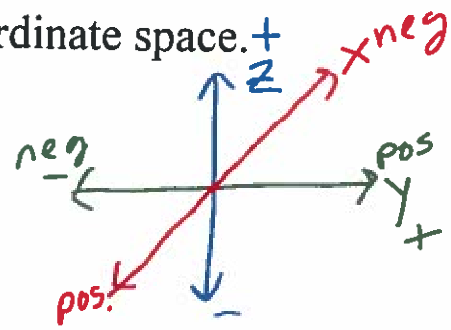
right or left

Describe the location in coordinate space. +

ex. (2, 3, 4) 2 F, 3R, 4 up

ex. (-5, 1, 0)

5 Backwards
1 right



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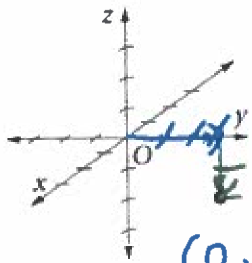
1 EXAMPLE Graphing in Coordinate Space

Graph each point in coordinate space.

a. (0, 3, −2)

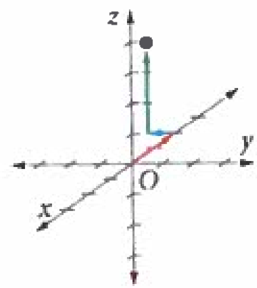
Sketch the axes. From the origin, move right 3 units and down 2 units.



(0,3,-2)

b. (−2, −1, 3)

Sketch the axes. From the origin, move back 2 units, left 1 unit, and up 3 units.



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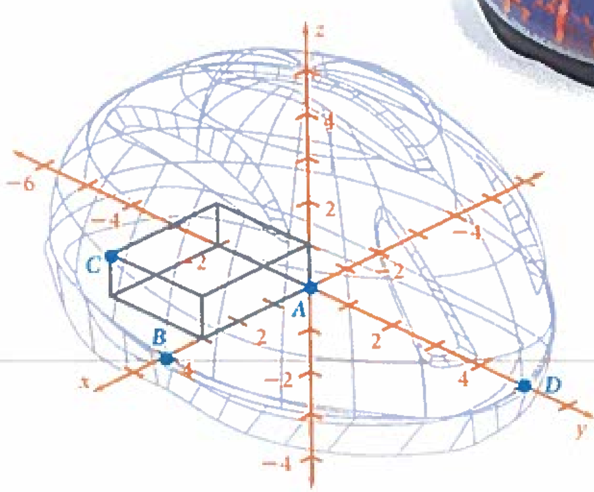
2 EXAMPLE Real-World Connection

Product Design Computers are used to design three-dimensional objects. Programs allow the designer to view the object from different perspectives. Find coordinates for points A, B, and C in the diagram below.



$A(0, 0, 0)$, $B(4, 0, 0)$, $C(3, -2, 1)$

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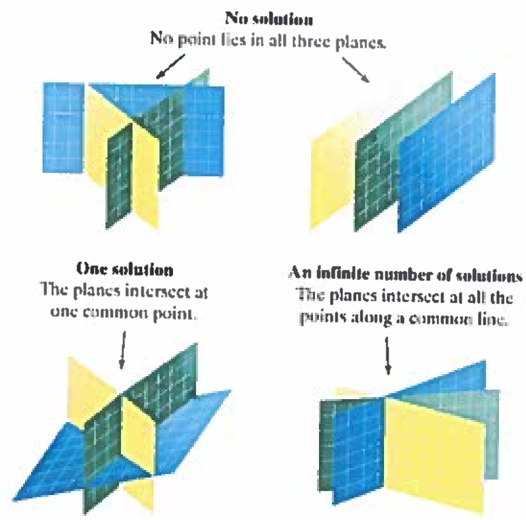
3-6 Systems With Three Variables

1 Solving Three-Variable Systems by Elimination

You can represent systems of equations in three variables as graphs in three dimensions. As you learned in Lesson 3-5, the graph of any equation of the form $Ax + By + Cz = D$, where A, B, and C are not all zero, is a plane. The solutions of a three-variable system can be shown graphically as the intersections of planes.



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When the solution of a system of equations in three variables is represented by one point, you can write it as an ordered triple (x, y, z).

You can solve a system of three equations in three variables by working with the equations in pairs. You will use one of the equations *twice*.

Solving Systems in Triangular Form ✱ Easiest

$x - 2y + 2z = 9$

$y + 2z = 5$

$z = 3$

$y + 2(3) = 5$

$y + 6 = 5$

$-6 \quad -6$

$y = -1$

$x - 2(-1) + 2(3) = 9$

$x + 2 + 6 = 9$

$x + 8 = 9$

$-8 \quad -8$

$\boxed{x = 1}$

$(1, -1, 3)$
x, y, z

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3. $\begin{cases} 2x - y + z = -2 \\ x + 3y - z = 10 \\ x + 2z = -8 \end{cases}$

Add

$3x + 2y = 8$ $\times -1$

$2x + 6y + 2z = 20$

$1x + 2z = -8$

$3x + 6y = 12$

$-3x - 2y = -8$

$3x + 6y = 12$

$4y = 4$

$\frac{4y}{4} = \frac{4}{4}$

$\boxed{y = 1}$

$3x + 6(1) = 12$

$3x + 6 = 12$

$-6 \quad -6$

$3x = 6$

$x = 2$

$2 + 2z = -8$

$-2 \quad -2$

$\frac{2z}{2} = \frac{-10}{2}$

$\boxed{z = -5}$

$\boxed{(2, 1, -5)}$

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21. A change machine contains nickels, dimes, and quarters. There are 75 coins in the machine, and the value of the coins is \$7.25. There are 5 times as many nickels as dimes. Find the number of coins of each type in the machine.

$$\begin{cases} N + D + Q = 75 \\ .05N + .10D + .25Q = 7.25 \\ 5N = D \end{cases}$$

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Substitution

$$\begin{cases} 2\ell + 2w + h = 72 \\ \ell = 3w \\ h = 2w \end{cases}$$

$2(3w) + 2w + 2w = 72$

$6w + 2w + 2w = 72$

$\frac{10w}{10} = \frac{72}{10}$

$\boxed{w = 7.2}$

$\ell = 3(7.2)$

$\boxed{\ell = 21.6}$

$h = 2(7.2)$

$\boxed{h = 14.4}$

(h, ℓ, w)

$(14.4, 21.6, 7.2)$

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46. What is the solution of the system? $\begin{cases} -3x + 2y - z = 6 \\ 3x + y + 2z = 5 \\ 2x - 2y - z = -5 \end{cases}$

x y z

A. (6, 5, −3)

x y z

B. (1, 4, −1)

$2(1) - 2(4) - (-1) = -5$

C. $\left(0, \frac{17}{5}, \frac{4}{5}\right)$

D. no solution

47. What is the solution of the system? $\begin{cases} x + 3y - 2z = -8 \\ 3x - y + z = 11 \\ 2x + 4y + 2z = 14 \end{cases}$

F. (2, 0, 5)

G. (−8, 11, 14)

H. $\left(-2, \frac{4}{3}, 5\right)$

J. no solution

Plug + chug

48. What is the solution of the system? $\begin{cases} y = -2x + 10 \\ -x + y - 2z = -2 \\ 3x - 2y + 4z = 7 \end{cases}$

A. $\left(3, -4, \frac{3}{2}\right)$

B. $\left(3, 16, \frac{15}{2}\right)$

C. $\left(-3, 16, \frac{15}{2}\right)$

D. $\left(3, 4, \frac{3}{2}\right)$

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Solve each system.

43. $\begin{cases} x + y + z = 10 \\ 2x - y + z = 2 \\ -x + 2y - z = 5 \end{cases}$

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