Day 38

4.5/4.6 Inverses

Part (1) If you multiply 2 matrices and the product is the identity matrix, then the 2 matrices are Multiplicative Inverses.

2×2 Identity Matrix 3×3

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(ex) Determine if the matrices are multiplicative inverses.

(ex) $\begin{bmatrix} -3 & 4 \\ 6 & -8 \end{bmatrix}$ (A), $\begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix}$ (B)

$[A][B] = \begin{bmatrix} 23 & 8 \\ -46 & -16 \end{bmatrix}$

NO this NOT the identity matrix

(ex) $\begin{bmatrix} 2 & 0.5 \\ 5 & 1 \end{bmatrix}$ (C), $\begin{bmatrix} -2 & 1 \\ 10 & -4 \end{bmatrix}$ (D)

$[C][D] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

$[D][C] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ✓

YES

Because Both products = identity matrix

PART 2 To Find Inverses By Hand

A^{-1} $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $A^{-1} = \frac{1}{\det[A]} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

ex $\begin{bmatrix} 6 & 2 \\ 3 & 1 \end{bmatrix}$

$\det = 6(1) - 3(2)$

$6 - 6$

$\det = 0$

NO Inverse

ex $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ $\det = 2(3) - 1(4)$

$6 - 4$

$\det = 2$

$A^{-1} = \frac{1}{2} \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1.5 & -2 \\ -.5 & 1 \end{bmatrix}$

PART 3 Solving Matrix Equations using Inverses

$[A]X = [B]$

$X = [A]^{-1} \cdot [B]$

LEFT

$[A]\begin{bmatrix} x \\ y \end{bmatrix} = [B]$

$\begin{bmatrix} x \\ y \end{bmatrix} = [A]^{-1} \cdot [B]$

ex $\underbrace{\begin{bmatrix} 3 & -4 \\ 4 & -5 \end{bmatrix}}_{A} X = \underbrace{\begin{bmatrix} 0 & -22 \\ 0 & -28 \end{bmatrix}}_{B}$

$X = [A]^{-1} \cdot [B]$

$X = \begin{bmatrix} -5 & 4 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 0 & -22 \\ 0 & -28 \end{bmatrix}$

$X = \begin{bmatrix} 0 & -2 \\ 0 & 4 \end{bmatrix}$

④ Change Systems of Eqs. into Matrix Eqs. and Solve by Inverses.

13. $\begin{cases} 6x + 9y = 36 \\ 4x + 13y = 2 \end{cases}$

Matrix Eq: $\begin{bmatrix} 6 & 9 \\ 4 & 13 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 36 \\ 2 \end{bmatrix}$

coefficient variable constant

1. Solve. $\begin{cases} x + y + z = 0.621 \\ 3x - 3y + 2z = -.007 \\ 4x + 5y - 10z = 1.804 \end{cases}$

MATRIX EQ

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & -3 & 2 \\ 4 & 5 & -10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} .621 \\ -.007 \\ 1.804 \end{bmatrix}$$

A B

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = [A^{-1}] \cdot [B]$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} .251 \\ .3 \\ .07 \end{bmatrix}$$

Name Block Date

Practice 4-5

2 × 2 Matrices, Determinants, and Inverses

Find the matrix E^{-1} for each. inverse

1. $E = \begin{bmatrix} 2 & -2 \\ -1 & 2 \end{bmatrix}$

2. $E = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

3. $E = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$

2

Find the inverse of each matrix, if it exists. If it does not exist, write *no inverse* and explain why not.

7. $\begin{bmatrix} 3 & 4 \\ -3 & 4 \end{bmatrix}$

8. $\begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

10. $\begin{bmatrix} 30 & -4 \\ -25 & 3 \end{bmatrix}$

Solve each matrix equation.

3

11. $\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} X = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

13. $\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix} X = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$

Determine whether the matrices are multiplicative inverses.

1

20. $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}, \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

21. $\begin{bmatrix} 4 & 9 \\ 2 & 6 \end{bmatrix}, \begin{bmatrix} 1 & -\frac{3}{2} \\ -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$

22. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

Practice 4-6

3 × 3 Matrices, Determinants, and Inverses

Where necessary, use a graphing calculator. Find the inverse (A^{-1}) of each matrix, if it exists. If it does not exist, write *no inverse*.

2

1. $\begin{bmatrix} 1 & 2 & 0 \\ -2 & 0 & -3 \\ 3 & -1 & 5 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 0 & 2 & 3 \end{bmatrix}$

Solve each equation for X.

9. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} X = \begin{bmatrix} 4 \\ -5 \\ 3 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 2 & 0 \\ -2 & 0 & -3 \\ 3 & -1 & 5 \end{bmatrix} X = \begin{bmatrix} -1 \\ 12 \\ -20 \end{bmatrix}$

Determine whether the matrices are multiplicative inverses.

18. $A = \begin{bmatrix} -2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{bmatrix}, B = \begin{bmatrix} -\frac{4}{3} & -\frac{5}{3} & 1 \\ -\frac{4}{3} & -\frac{8}{3} & 1 \\ 1 & \frac{2}{3} & 0 \end{bmatrix}$

19. $A = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}$

Write each system as a matrix equation. Identify the coefficient matrix, the variable matrix, and the constant matrix.

13. $\begin{cases} 6x + 9y = 36 \\ 4x + 13y = 2 \end{cases}$

14. $\begin{cases} 3x - 4y = -9 \\ 0x + 7y = 24 \end{cases}$

15. $\begin{cases} 4x + 0y - 1z = 9 \\ 12x + 2y + 0z = 17 \\ 1x - 1y + 12z = 3 \end{cases}$

Solve each matrix equation. If the coefficient matrix has no inverse, write *no unique solution*.

18. $\begin{bmatrix} 0.25 & -0.75 \\ 3.5 & 2.25 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1.5 \\ -3.75 \end{bmatrix}$

19. $\begin{bmatrix} 3 & -9 \\ 1 & -6 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \end{bmatrix}$

Solve each system.

1. $\begin{cases} x + y + z = 0.621 \\ 3x - 3y + 2z = -0.007 \\ 4x + 5y - 10z = 1.804 \end{cases}$

5. $\begin{cases} 4x + y + z = 0 \\ 5x + 2y + 3z = -15 \\ 6x - 5y - 5z = 52 \end{cases}$

6. $\begin{cases} 2x + 3y = 12 \\ x + 2y = 7 \end{cases}$

8. $\begin{cases} x - 3y = -1 \\ -6x + 19y = 6 \end{cases}$