Day 40

11.3 Geometric Sequences

A sequence whose consecutive terms have a **common ratio**, is **geometric**.

* you multiply or divide to get the next term in the sequence

(ex) 5, 15, 45, 135, ...

$\frac{15}{5} = 3 \quad \frac{45}{15} = 3 \quad \frac{135}{45} = 3$

common ratio: $r = 3$ ← integer # multiplication

is geometric

if the ratio is a fraction it's division or multiplication of a fraction.

(ex) 15, 30, 45, 60, ...

$\frac{30}{15} = 2 \quad \frac{45}{30} = 1.5$: NOT geometric

$30 - 15 = 15 \quad 45 - 30 = 15 \quad 60 - 45 = 15$

Arithmetic
$d = 15$

p. 613 Quick V #1

Arithmetic, Geometric, or Neither?

b) 6, −24, 96, −384, ... → geometric, $r = -4$
alternating signs

c.) 8, 20, 32, 44, ... arithmetic
$d = 12$

Geometric Sequence Formulas

Recursive $a_n = a_{n-1} \cdot R$

Explicit $a_n = a_1 \cdot R^{n-1}$

a_1 = 1st term
R = common ratio

Quick ✓ #2 p.613

Find the 19th term. (Explicit)

a) 11, 33, 99, 297, ... $R = 3$ $a_1 = 11$

Geometric $\boxed{a_n = 11(3)^{n-1}}$ explicit formula

$a_{19} = 11(3)^{19-1}$ $a_{19} = 11(3)^{18}$

$a_{19} = 4,261,625,379$

b) 20, 17, 14, 11, 8, ... Arithmetic $d = -3$

$a_n = a_1 + (n-1)d$

$\boxed{a_n = 20 + (n-1)(-3)}$ explicit formula

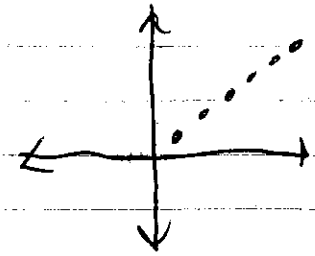
$a_{19} = 20 + (19-1)*-3$

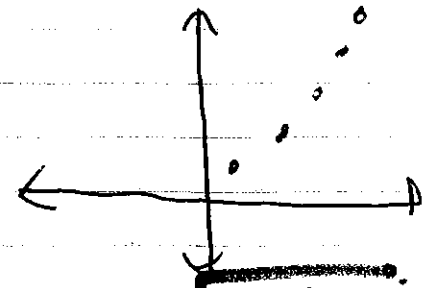
$\boxed{a_{19} = -34}$

Geometric Mean of any 2 positive #'s is taken by finding the positive square root of the product of the 2 #'s.

The graph is exponential.

(Arithmetic is linear)





$$g.m. = \sqrt{a \cdot b}$$

$$a, \underset{\pm}{\frac{\pm}{\uparrow}}, b$$

Quick V #3. Find the geometric mean.

a) 20, □, 80, ... $\sqrt{(20 \cdot 80)}$

$g.m. = 40$ or -40
$= \pm 40$

b.) 3, □, 18.75, ... $\sqrt{(3 * 18.75)}$

$g.m. = \pm 7.5$

Examples: p. 615

#15 $\boxed{a_1 = \frac{1}{2}}$ $r = \frac{2}{3}$

$a_n = a_1 r^{n-1}$

$a_n = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^{n-1}$

Explicit Formula
Find 1st 5 terms.

$a_2 = a_1 r^{2-1}$

$a_2 = a_1 r^1 = \frac{1}{3}$

$a_3 = 2/9$ $r_4 = \frac{4}{27}$ $r_5 = \frac{8}{81}$

#52) $a_9 = -\frac{1}{3}$ $r = -\frac{1}{6}$

Find the 10th term.

$a_n = a_{n-1} \cdot R$

$a_{10} = a_{10-1}$

$a_{10} = a_9 \cdot R$

$a_{10} = \left(-\frac{1}{3}\right)\left(-\frac{1}{6}\right)$ $\boxed{a_{10} = \frac{1}{18}}$

#53) $a_{11} = -\frac{1}{3}$ $r = \frac{1}{2}$

$a_{11} = a_{10} \cdot R$ ← recursive formula

$\frac{2}{1} \cdot -\frac{1}{3} = a_{10}\left(\frac{1}{2}\right) \cdot \frac{2}{1}$

$\boxed{\frac{-2}{3} = a_{10}}$

p. 614-615 (2-42 Even, 42-44, 48-51)