

## 10.5 Hyperbolas

A hyperbola is a set of points  $P$  in a plane such that the abs. value of the difference between the distances from  $P$  to 2 fixed points  $F_1$  and  $F_2$  is a constant.

$$|PF_1 - PF_2| = k, \quad k < F_1F_2$$

Each fixed point is a focus of the hyperbola.

The segment that lies on the line containing the foci and has endpoints on a hyperbola is the transverse axis (major).

The endpoints are the vertices.

The midpoint of the segment connecting the vertices is the center.

Standard Forms of the Equation for a Hyperbola centered at  $(0,0)$

①

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

vertices:  $(\pm a, 0)$

asymptotes:  $y = \pm \frac{b}{a}x$

Foci:  $(\pm c, 0)$

x-intercepts:  $\pm a$

NO y-intercepts

horizontal transverse axis is the "x"

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$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

vertical  
transverse  
axis = y

vertices:  $(0, \pm a)$

Foci:  $(0, \pm c)$

asymptotes:  $y = \pm \frac{a}{b} x$

no x-intercepts

y-intercepts:  $\pm a$

Foci equation:  $c^2 = a^2 + b^2$

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Examples:

1)

Graph.  $\frac{x^2}{16} - \frac{y^2}{4} = 1$

$$a^2 = 16$$
$$a = \pm 4$$

$$b^2 = 4$$
$$b = \pm 2$$

$$c^2 = 16 + 4$$
$$c^2 = 20$$
$$c = 2\sqrt{5}$$

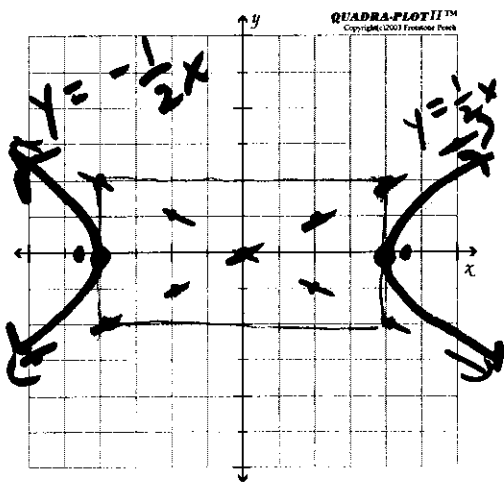
vertices:  $(\pm 4, 0)$

Foci:  $(\pm 2\sqrt{5}, 0)$

asymptotes:

$$y = \pm \frac{b}{a} x \quad y = \pm \frac{2}{4} x$$

$$y = \pm \frac{1}{2} x$$



ex 2 Graph.

$$\frac{81y^2}{729} - \frac{9x^2}{729} = \frac{729}{729}$$

$$\frac{y^2}{9} - \frac{x^2}{81} = 1$$

$$a = \pm 3 \quad b = \pm 9$$

$$c^2 = 9 + 81$$

$$c^2 = 90$$

$$c = \pm 3\sqrt{10}$$

vertices  $(0, \pm 3)$

foci  $(0, \pm 3\sqrt{10})$

asymptotes:  $y = \pm \frac{a}{b}x$

$$y = \pm \frac{3}{9}x$$

$$y = \pm \frac{1}{3}x$$

ex 3 Find the eq. Assume the transverse axis is horizontal.  
(x)

$$a = 263$$

$$c = 407$$

$$\boxed{\frac{x^2}{69169} - \frac{y^2}{96480} = 1}$$

$$a^2 + b^2 = c^2$$

$$263^2 + b^2 = 407^2$$

$$b^2 = 96480$$

ex 4 foci  $(0, \pm 2)$  vertices  $(0, \pm 1)$

$$c = 2$$

$$a = 1$$

$$2^2 = 1^2 + b^2$$

$$3 = b^2$$

$$\boxed{\frac{y^2}{1} - \frac{x^2}{3} = 1}$$

ex 4 Write the eq. of a hyperbola with the given foci and vertices

a) foci  $(\pm 5, 0)$  vertices  $(\pm 3, 0)$

b.) foci  $(0, \pm 2)$  vertices  $(0, \pm 1)$

$$\begin{aligned}x^2 - 2y^2 &= 4 \\ \frac{-2y^2}{-2} &= \frac{-x^2 + 4}{-2} \\ y^2 &= \frac{1}{2}x^2 - 2\end{aligned}$$

$$y = \pm \sqrt{\left(\frac{1}{2}x^2 - 2\right)}$$

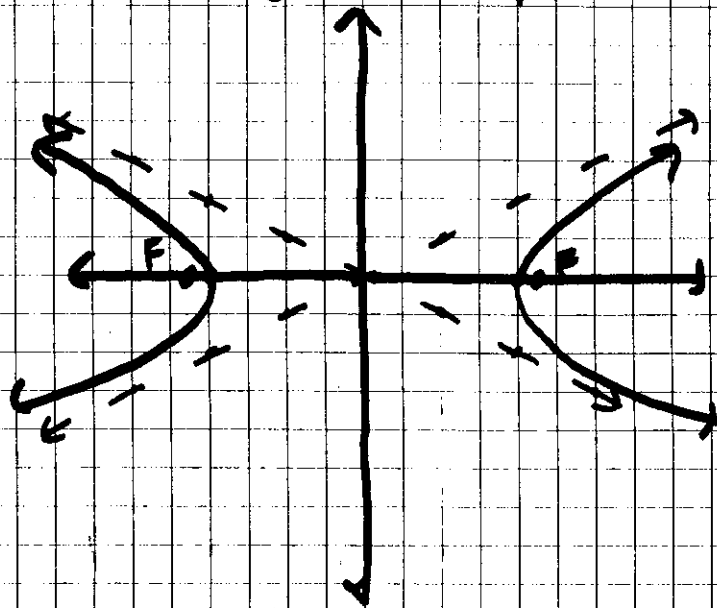
$$\begin{aligned}81y^2 &= 9x^2 + 729 \\ y^2 &= \frac{1}{9}x^2 + 81\end{aligned}$$

$$y = \sqrt{\frac{1}{9}x^2 + 81}$$

# Hyperbolas

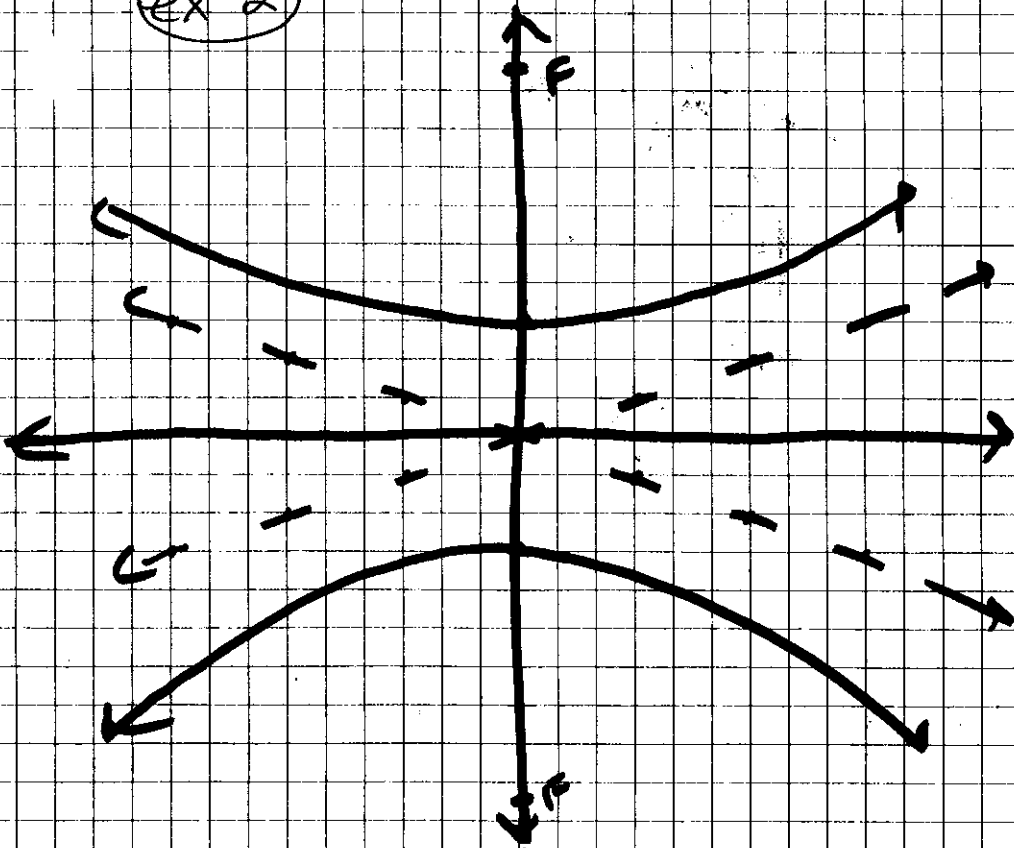
ex 1

$$\frac{x^2}{16} - \frac{y^2}{4} = 1$$



ex 2

$$81y^2 - 9x^2 = 729$$



Graph.

$$\frac{9x^2}{441} - \frac{49y^2}{441} = \frac{441}{441}$$

$$\frac{x^2}{49} - \frac{y^2}{9} = 1$$

$$a = \pm 7 \quad b = \pm 3$$

asymptotes:  $y = \frac{b}{a}x$

$$y = \pm \frac{3}{7}x$$

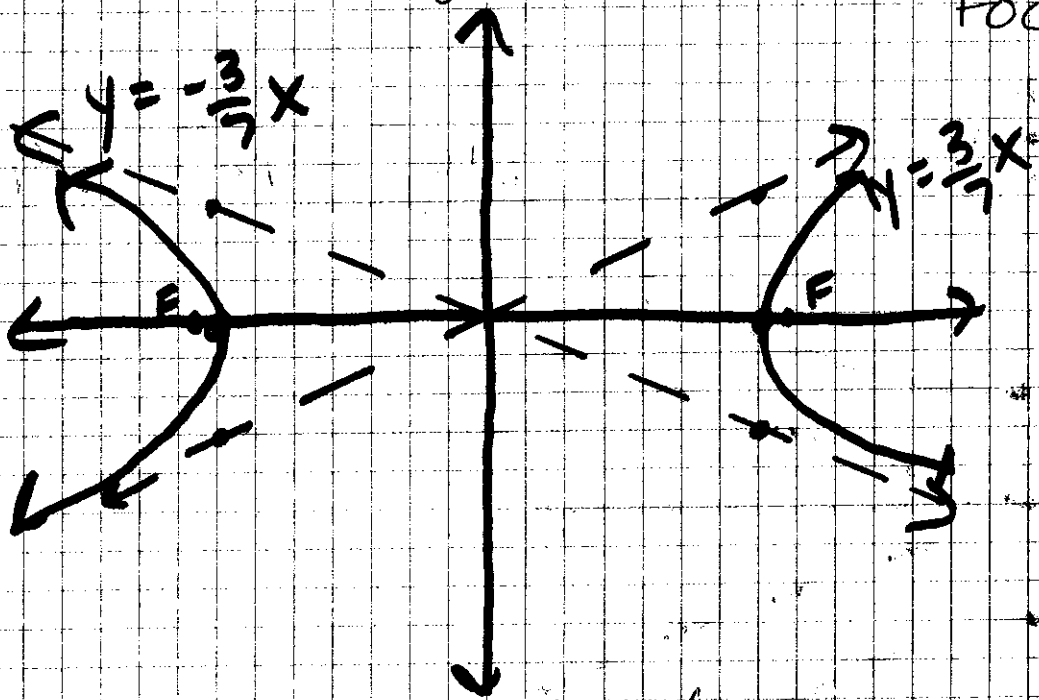
$$c^2 = 49 + 9$$

$$c^2 = 58$$

$$c = \pm\sqrt{58}$$

vertices:  $(\pm 7, 0)$

Foci:  $(\pm\sqrt{58}, 0)$



p. 578 (2-12 even, 20-26 even)