11.1 Mathematical Patterns

You can describe some patterns with a <u>sequence</u> or ordered list of the #'s.
- each # in a sequence is a <u>term</u>.

Any sequence of #'s that increase:
the sequence's pattern will be either adding, multiplying, cubing, squaring, ...

ex $\underset{a_1}{27}, \underset{a_2}{34}, \underset{a_3}{41}, \underset{a_4}{48}, \ldots$

rule: $\boxed{\text{add } 7}$

$34 - 27 = 7$
$41 - 34 = 7$
$48 - 41 = 7$

term − previous term
$a_2 - a_1 \quad a_3 - a_2$

ex 1, 4, 16, 64... Koch snowflake

$4 \div 1 = 4$
$16 \div 4 = 4$
$64 \div 4 = 4$

$\boxed{\text{rule: multiply by } 4}$

term ÷ previous term

ex 1, 1, 2, 3, 5, 8, ... Fibonacci

adding the 2 previous #'s to get the next term.

ex 1, 4, 9, 16, 25, ... n^2 rule

$4 - 1 = \cancel{3}$ $\quad 4 \div 1 = 4 \quad 1^2 = 1 \quad 3^2 = 9$
$9 - 4 = \cancel{5}$ $\quad 9 \div 4 = \frac{9}{4} = 2\frac{1}{4} \quad 2^2 = 4$

Any sequence that decreases will have a rule of subtraction, division, multiplication of a # less than 1, ...

ex 243, 81, 27, 9, ... $\boxed{\text{rule: } \div \text{ by } 3}$

$243 \div 81 = 3$ $\quad$ $81 \div 27 = 3$

1st ÷ 2nd $\quad$ $27 \div 9 = 3$

previous term ÷ next term

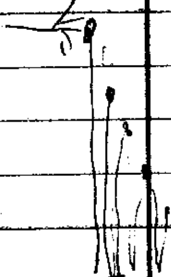
ex $5, \frac{9}{2}, 4, \frac{7}{2}, 3, \frac{5}{2}, \ldots$ $\boxed{\text{rule: subtract } \frac{1}{2}}$

$\frac{9}{2} = 4.5 \quad \frac{7}{2} = 3.5$

$5 - 4.5 = .5$

$4.5 - 4 = .5$

ex Drop a ball from a height of 10ft. After the ball hits the floor, it rebounds to 85% of its previous height. About how high will it rebound after its 4th bounce.



10 ft

after 1st bounce $10(.85) = 8.5$ ft

2nd bounce $8.5(.85) = 7.225$ ft

3rd " $7.225(.85) = 6.141$ ft

4th $6.14125(.85) = 5.220$ ft

You can use a variable, such as "a", with positive integer subscripts to represent the terms in the sequence.

1st	2nd	3rd ...	n-1 term	n^{th}	n+1
a_1	a_2	a_3	a_{n-1} previous	a_n	a_{n+1} after

A recursive formula defines the terms in a sequence by relating each term to the one before it.

The bouncing ball problem is recursive because the height of the ball after each bounce was 85% of the previous height.

The formula is $\boxed{a_n = .85a_{n-1}}$

ex 3 p. 602

$2, 4, 6, 8, 10, \ldots \underline{12}, \underline{14}, \underline{16}$

rule, add 2

$\boxed{a_n = a_{n-1} + 2}$ where $a_1 = 2$

$a_5 = 10$ $\quad a_6 = a_{6-1} + 2$

$= a_5 + 2$

$= 10 + 2$

$a_6 = 12$

$a_7 = 14$

An explicit formula is a formula that expresses the n^{th} term in terms of n.

1st 2nd 3rd 4th 5th

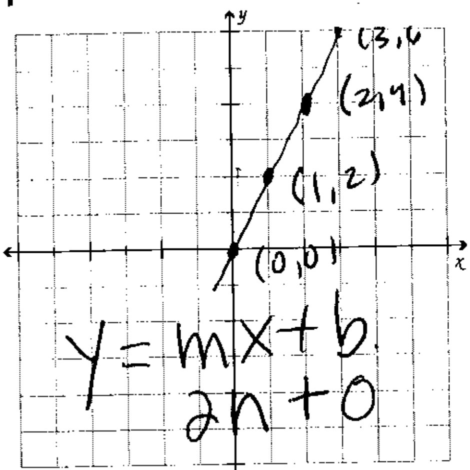
$\underline{0}, 2, 4, 6, 8, 10, \ldots$

2 2 2

~~n+2~~

explicit formula

$\boxed{2n}$



$y = mx + b$

$2n + 0$

Find the 1st term

$2(1) = 2$

3rd term $n = 3$

$2(3) = 6$

a_0 a_1 a_2 a_3 a_4 a_5 a_6

ex $\underline{-3}, 2, 7, 12, 17, 22, \ldots$

y-intercept 5 5 5 ← slope

$a_5 + 5 = 27$

$22 + 5$

recursive formula: $a_n = a_{n-1} + 5$

explicit formula: $5n - 3$

$5(6) - 3$

$30 - 3$

27

previous term = 22

ex 19, 16, 13, 10, 7, ...

-3 -3 -3 -3

decreasing by 3

recursive formula: $a_n = a_{n-1} - 3$

explicit formula: $-3n + 22$