

Day 78

14.1 Trig Identities

① $\frac{\sin \theta}{\cos \theta} = \tan \theta$
tangent identity

② $\frac{\cos \theta}{\sin \theta} = \cot \theta$
cotangent identity

Reciprocal Identities

① $\csc \theta = \frac{1}{\sin \theta}$ ② $\sec \theta = \frac{1}{\cos \theta}$

③ $\cot \theta = \frac{1}{\tan \theta}$

Pythagorean identities

① $\cos^2 \theta + \sin^2 \theta = 1$ $\begin{cases} \sin^2 \theta = 1 - \cos^2 \theta \\ \cos^2 \theta = 1 - \sin^2 \theta \end{cases}$

② $1 + \tan^2 \theta = \sec^2 \theta$ $\begin{cases} \tan^2 \theta = \sec^2 \theta - 1 \\ \sec^2 \theta - \tan^2 \theta = 1 \end{cases}$

③ $1 + \cot^2 \theta = \csc^2 \theta$ $\begin{cases} \cot^2 \theta = \csc^2 \theta - 1 \\ \csc^2 \theta - \cot^2 \theta = 1 \end{cases}$

Verify Identity

① $1 + \tan^2 \theta = \sec^2 \theta$

$\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta}$
 $\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta}$

$\frac{1}{\cos^2 \theta} = \sec^2 \theta$

$$(2) \quad 1 + \cot^2 \theta = \csc^2 \theta$$

$$\frac{1 + \frac{\cos^2 \theta}{\sin^2 \theta}}{1} = \csc^2 \theta$$

$$\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \csc^2 \theta$$

$$\frac{1}{\sin^2 \theta} = \csc^2 \theta$$

$$\csc^2 \theta = \csc^2 \theta \quad \checkmark$$

$$(3) \quad \tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta$$

$$\frac{\sin^2 \theta}{\cos^2 \theta} - \frac{\sin^2 \theta \cdot \cos^2 \theta}{1 \cdot \cos^2 \theta} = \underline{\tan^2 \theta \sin^2 \theta}$$

$$\frac{\sin^2 \theta}{\cos^2 \theta} - \frac{\sin^2 \theta \cos^2 \theta}{\cos^2 \theta} = 11$$

$$\frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta} = 11$$

Factor
out
GCF

$$\frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta} = 11$$

$$\tan^2 \theta \sin^2 \theta = 11$$

$$\begin{aligned} \cancel{\cos^2 \theta} + \sin^2 \theta &= 1 \\ -\cos^2 \theta & \\ \hline \sin^2 \theta &= 1 - \cos^2 \theta \end{aligned}$$

$$\textcircled{4} \sec^2 \theta - \sec^2 \theta \cos^2 \theta = \tan^2 \theta$$

Factor
out
GCF

$$\sec^2 \theta (1 - \cos^2 \theta) = \tan^2 \theta$$

$$\underline{\sec^2 \theta} (\sin^2 \theta) = \tan^2 \theta$$

$$\left(\frac{1}{\cos^2 \theta} \right) \cdot \sin^2 \theta = \tan^2 \theta$$

$$\frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta$$

$$\tan^2 \theta = \tan^2 \theta \quad \checkmark$$

Simplifying Trig Expressions

$$\textcircled{1} \csc \theta \tan \theta = \frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta}$$

$$= \frac{1}{\cos \theta}$$

$$= \boxed{\sec \theta}$$

$$\textcircled{2} \sec \theta \cot \theta = \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta}$$

$$= \frac{1}{\sin \theta}$$

$$= \boxed{\csc \theta}$$

$$\textcircled{3} (1 + \cot^2 \theta)(\sec^2 \theta - 1)$$

$$(\csc^2 \theta)(\tan^2 \theta)$$

$$\frac{1}{\cancel{\sin^2 \theta}} \cdot \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$\frac{1}{\cos^2 \theta}$$

$$\boxed{\sec^2 \theta}$$

$\textcircled{4}$

$$(1 - \sec \theta)(1 + \sec \theta)$$

(multiply)
FOIL

$$1 + \cancel{\sec \theta} - \cancel{\sec \theta} - \sec^2 \theta$$

$$1 - \sec^2 \theta$$

$$-1(-1 + \sec^2 \theta)$$

$$-1(\sec^2 \theta - 1)$$

$$\boxed{-1 \tan^2 \theta}$$

p. 780 - 781 (2-38 Even)