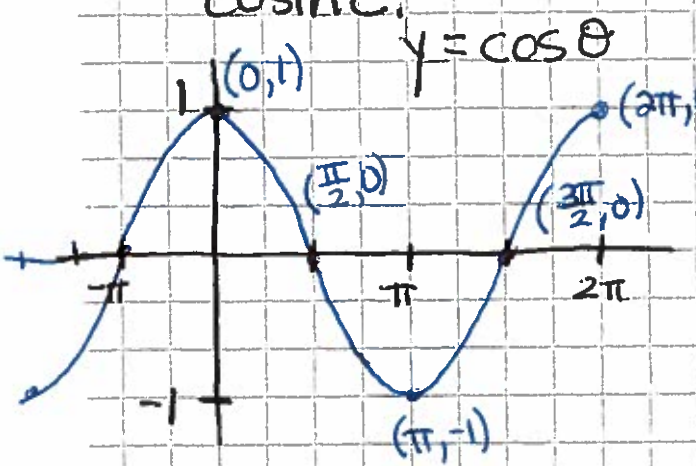
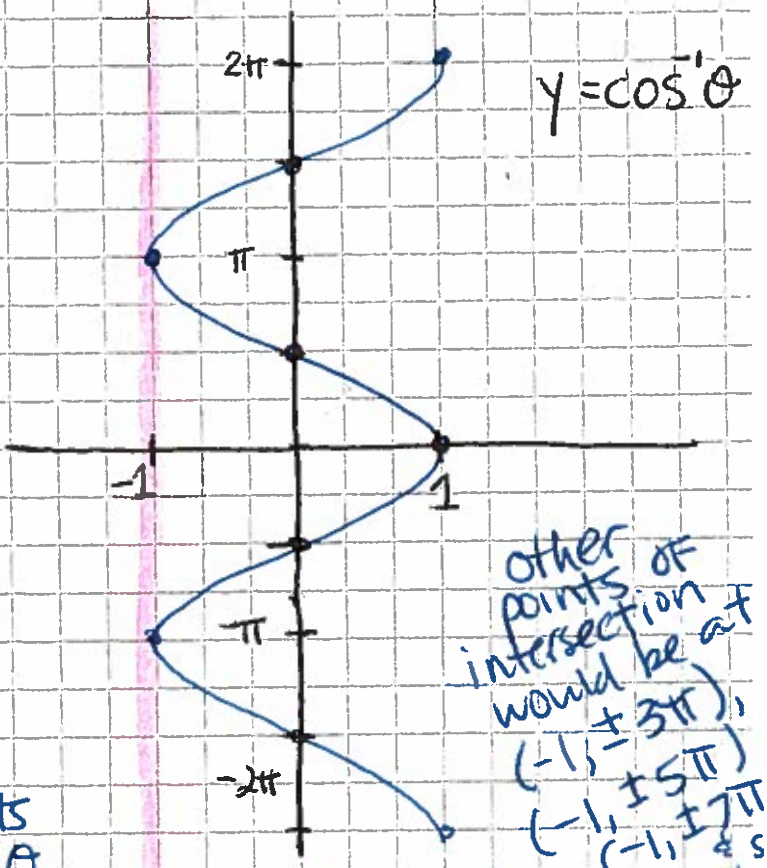
14.2 Notes... PART 2

Day 79

① Using a graph to find ∠'s with the given cosine.





a) Find the radian measures whose cosine is -1

Draw the line $x = -1$ see where it intersects the graph of $y = \cos^{-1}\theta$

it intersects at $(-1, \pi)$ and $(-1, -\pi)$

other points of intersection would be at $(-1, \pm 3\pi)$, $(-1, \pm 5\pi)$ $(-1, \pm 7\pi)$ etc

the distance between those is 2π so the measures of all ∠'s can be written $\pi + 2\pi n$

b) Find the radian measures of the angles θ whose cosine is 2.

The line $x = 2$ would not intersect the graph so there is no solution

c) $\cos\theta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$ or $\frac{\pi}{2} + \pi n$

d) $\cos -1$. none

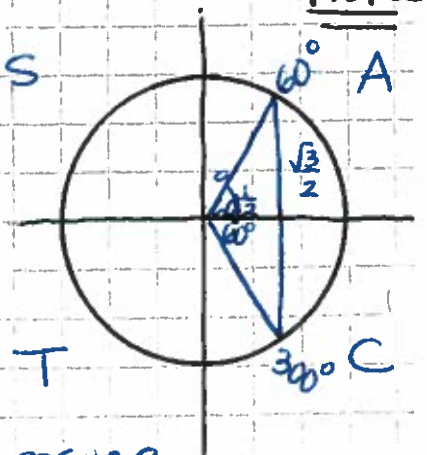
You can use a unit circle to find angle measures for given sine or cosine values.

2 EXAMPLE **Using a Unit Circle**

Multiple Choice Find the degree measures of the angles whose cosine is $\frac{1}{2}$. $(x = \frac{1}{2})$

(A) $30° + n \cdot 360°$ and $330° + n \cdot 360°$ (B) $60° + n \cdot 360°$ and $300° + n \cdot 360°$

(C) $30° + n \cdot 360°$ and $210° + n \cdot 360°$ (D) $60° + n \cdot 360°$ and $120° + n \cdot 360°$



Cosine is positive in quadrant I & IV

so solution is B; Period for cosine is 2π radians or 360°

❷ Use a unit circle. Find the degree measures of all angles with the given cosine.

a. $-\frac{1}{2}$ b. $-\frac{\sqrt{3}}{2}$ c. $\frac{\sqrt{2}}{2}$

degrees

$120^\circ + n \cdot 360^\circ$ $150^\circ + 360n$ $45^\circ + 360n = \frac{\pi}{4} + 2\pi n$

$240^\circ + 360^\circ n$ $210^\circ + 360n$ $315^\circ + 360n = \frac{7\pi}{4} + 2\pi n$

or $\frac{5\pi}{6} + 2\pi n$ & $\frac{7\pi}{6} + 2\pi n$

3 EXAMPLE Using a Calculator to Find the Inverse of Sine

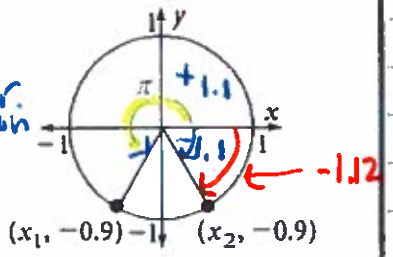
Use a calculator and an inverse function to find the radian measures of all the angles whose sine is -0.9.

$\sin^{-1}(-0.9) \approx -1.12$ Use a calculator.

This angle is in Quadrant IV. The sine function is also negative in Quadrant III, as shown in the figure at the right. So $\pi + 1.12 = 4.26$ is another solution.

The radian measures of all the angles whose sine is -0.9 can be written as

$-1.12 + 2\pi n$ and $4.26 + 2\pi n$



Quick Check

❸ Find the radian measures of the angles.

a. angles whose sine is 0.44

$.46 + 2\pi n$ and $2.69 + 2\pi n$

b. angles whose sine is -0.73

$-0.82 + 2\pi n$ and $3.96 + 2\pi n$

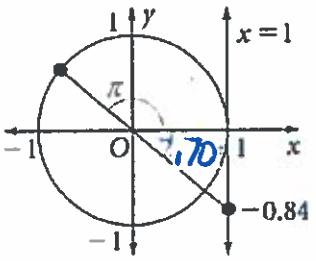
4 EXAMPLE Using a Calculator to Find the Inverse of Tangent

Use a calculator and an inverse function to find the measure in radians of all the angles whose tangent is -0.84.

$\tan^{-1}(-0.84) \approx -.70$ Use a calculator.

The tangent function is also negative in Quadrant , as shown in the figure at the right. So $\pi -$ $\approx$ is another solution.

The radian measures of all the angles whose tangent is -0.84 can be written as



Period for tangent is π

Quick Check

❹ Find the radian measures of the angles.

a. angles whose tangent is 0.44

$\tan^{-1}(.44) = .41$

$.41 + \pi n$

b. angles whose tangent is -0.73

$\tan^{-1}(-.73) = -.63$

$-.63 + \pi n$

14.2 notes...

Solve each equation for $0 \le \theta < 2\pi$

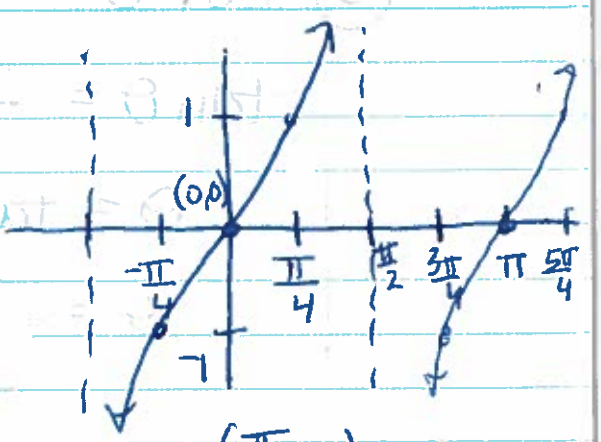
(5) $4\tan\theta = 3 + \tan\theta$

$-\tan\theta \quad -\tan\theta$

$\frac{3\tan\theta}{3} = \frac{3}{3}$

$\tan\theta = 1$

$\theta = .785$ or $\frac{\pi}{4}$ ← $\tan^{-1}(1)$



at $(\frac{\pi}{4}, 1)$

$\frac{\pi}{4} + \pi = \frac{5\pi}{4}$

$\frac{5\pi}{4} + \pi = \frac{9\pi}{4}$ ← not in solution range so

the solutions are ($\frac{\pi}{4}$ and $\frac{5\pi}{4}$)

(6) $\tan^2\theta + \tan\theta = 0$

$\tan\theta(\tan\theta + 1) = 0$

$\tan\theta = 0 \quad \tan\theta = -1$

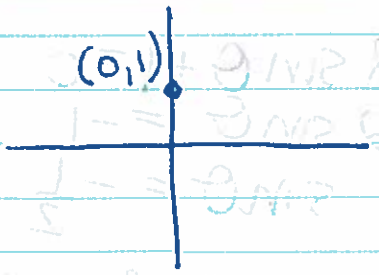
$\theta = 0 \quad \theta = \frac{-\pi}{4} + \pi = \frac{-\pi}{4} + \frac{4\pi}{4} = \frac{3\pi}{4}$

$\theta = \frac{3\pi}{4} + \pi = \frac{7\pi}{4}$

(7) $\sec\theta = \frac{1}{1}$

$\cos\theta = \frac{1}{1}$

$\theta = 0$



(8) $\cot\theta = -10$

$\tan\theta = \frac{-1}{10}$

$\theta = \tan^{-1}\left(\frac{1}{10}\right)$

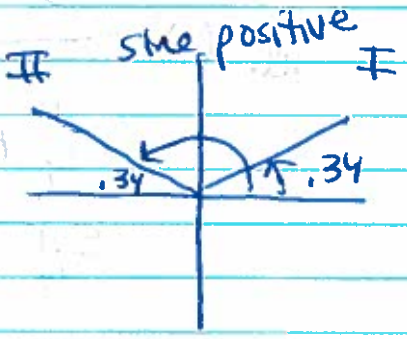
$\theta = 3.04$ & $3.04 + \pi = 6.18$

(9) $\csc\theta = 3$

$\sin\theta = \frac{1}{3}$

$\theta = \sin^{-1}\left(\frac{1}{3}\right) = .34$

$\theta = \pi - .34 = 2.80$



(10) Solve

$2\sin^2\theta - 3\sin\theta = -2$

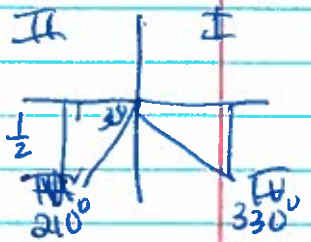
$2\sin^2\theta - 3\sin\theta + 2 = 0$

$(2\sin\theta + 1)(\sin\theta - 2) = 0$

$2\sin\theta + 1 = 0$ $\quad$ $\sin\theta - 2 = 0$

$2\sin\theta = -1$ $\quad$ $\sin\theta = 2$

$\sin\theta = -\frac{1}{2}$



$210° + 330°$

$\frac{7\pi}{12}$ and $\frac{11\pi}{12}$