10-1 Exploring Conic Sections

Check Skills You'll Need — GO for Help Lessons 2-2, 5-2, and 5-5

Find the x- and y-intercepts of the graph of each function.

1. $y = 3x + 6$

$y = mx + b$

y-int: 6

$0 = 3x + 6$

$-6 \quad -6$

$\frac{-6}{3} = \frac{3x}{3}$

$-2 = x$

2. $2y = -x - 3$

$2 \cdot 0 = -x - 3$

$0 = -x - 3$

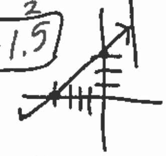
$+x \quad +x$

$\boxed{x = -3}$

$2y = - \quad - 3$

$\frac{2y}{2} = \frac{-3}{2}$

$\boxed{y = -1.5}$



3. $3x - 4y = -12$

$3x - 4 \cdot 0 = -12$

$\frac{3x}{3} = \frac{-12}{3}$

$\boxed{x = -4}$

$3 \cdot 0 - 4y = -12$

$0 - 4y$

$\frac{-4y}{-4} = \frac{-12}{-4}$

$y = 3$

4. $y = x^2 - 4$

$0 = x^2 - 4$

$+4 \quad +4$

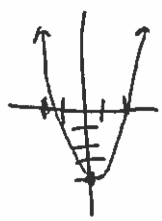
$\sqrt{4} = \sqrt{x^2}$

$\pm 2 = x$

$y = 0^2 - 4$

$y = -4$

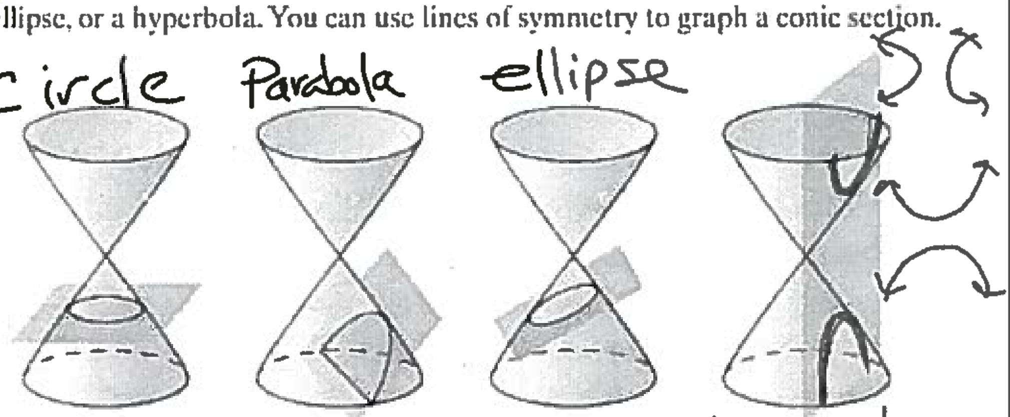
5. $y = (x - 3)^2$



6. $y = -4x^2 + 1$

A **conic section** is a curve formed by the intersection of a plane and a double cone. By changing the inclination of the plane, you can create a circle, a parabola, an ellipse, or a hyperbola. You can use lines of symmetry to graph a conic section.

circle Parabola ellipse

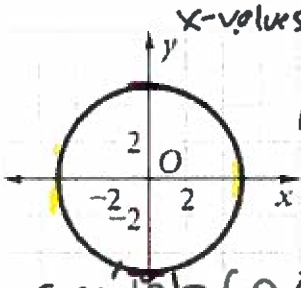


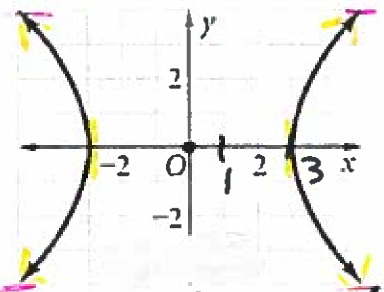
Hyperbola

Describe the graph and its lines of symmetry. → x-axis & y-axis

Then find the domain and range.

x-values y-values





A circle Has an infinite amount of Lines of Symmetry

center (0,0)
R = 5

D: [-5, 5]
OR $-5 \le x \le 5$

R: [-5, 5] OR $-5 \le y \le 5$

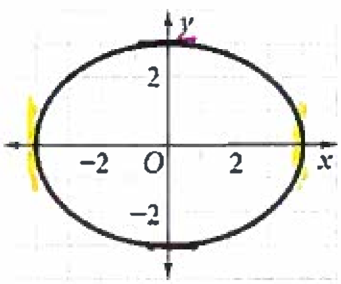
D: $x \le -3$ and $x \ge 3$

OR $(-\infty, -3] \cup [3, \infty)$

R: $(-\infty, \infty)$
OR All Real #'s

Describe the graph and its lines of symmetry. x-axis, y-axis

Then find the domain and range.



Domain

D(width): x-values

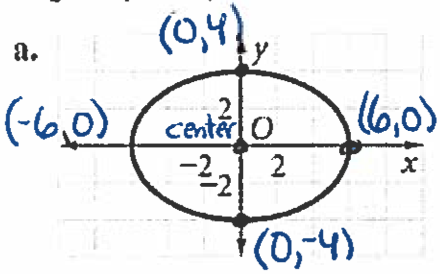
D: [-4, 4]

R: [-3, 3]

Range (Height) How tall?

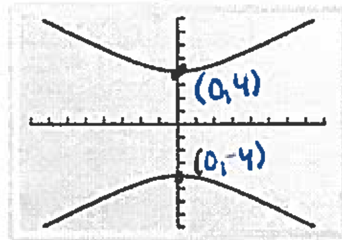
Identify the center and intercepts of each conic section. Then find the domain and range. In part (b), each interval on the graph represents one unit.

a.



D: [-6,6]
R: [-4,4]

b.



D: (never stops getting wider)
(-∞,∞)
R: (-∞,-4] ∪ [4,∞)

Parabolas: only the x or y value is squared, not both

ex $y=x^2+5$ OR $x+y^2=3$

Identify type of Conic:

Graphing Conic Sections

$1x^2+1y^2=9$ Circle — same coefficients

$3x^2+1y^2=9$ Ellipse — different coefficients

1st identify the x and y-intercepts.

Set $y = 0$, and solve for x.

Set $x = 0$, and solve for y.

It is possible it might NOT have both x & y intercepts.

Then - Solve the equation for y. Use the graphing calculator.

Put the positive equation in the $y_1 = \sqrt{}$

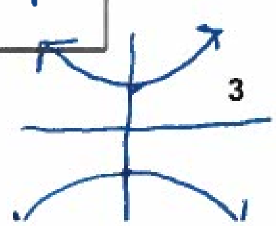
and the negative equation in the $y_2 = -\sqrt{}$

Use the table of values for both y_1 & y_2 to graph it.

Minus Sign: Hyperbola

$3x^2-y^2=9$ only has x-intercepts

$3y^2-x^2=9$ only has y-intercepts



Graph each equation. Identify the conic section and describe the graph and its lines of symmetry. Then find the domain and range.

$\boxed{4.\ 3y^2 - x^2 = 9}$

Find Intercepts:

$3 \cdot 0^2 - x^2 = 9$

$\frac{-1x^2}{-1} = \frac{9}{-1}$

$\sqrt{x^2} = \sqrt{-9}$

NO x-intercepts

$3y^2 - 0^2 = 9$

$\frac{3y^2}{3} = \frac{9}{3}$

$\sqrt{y^2} = \sqrt{3}$

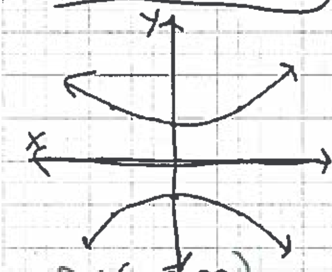
$y = \pm\sqrt{3}$ or ≈ 1.73

$3y^2 - x^2 = 9$
$\quad\ \ + x^2 \quad + x^2$

$\frac{3y^2}{3} = \frac{x^2}{3} + \frac{9}{3}$

$\sqrt{y^2} = \sqrt{\frac{x^2}{3} + 3}$

$y = \pm\sqrt{\frac{x^2}{3} + 3}$



D: $(-\infty, \infty)$

R: $(-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$

or $y \le -\sqrt{3}$ and $y \ge \sqrt{3}$

Graph each equation. Identify the conic section and describe the graph and its lines of symmetry. Then find the domain and range.

$\boxed{5.\ 4x^2 + 25y^2 = 100}$

1st: Find Intercepts!

$4x^2 + 25 \cdot 0^2 = 100$

$\frac{4x^2}{4} = \frac{100}{4}$

$\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$

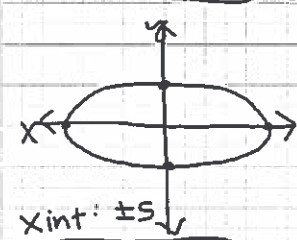
Ellipse

$\cancel{4 \cdot 0^2} + 25y^2 = 100$

$\frac{25y^2}{25} = \frac{100}{25}$

$\sqrt{y^2} = \sqrt{4}$

$y = \pm 2$



$4x^2 + 25y^2 = 100$
$-4x^2 \qquad\quad -4x^2$

$\frac{25y^2}{25} = \frac{100 - 4x^2}{25 \quad 25}$

$\sqrt{y^2} = \sqrt{4 - \left(\frac{4}{25}\right)x^2}$

$y = \pm\sqrt{4 - \left(\frac{4}{25}\right)x^2}$

xint: ± 5

D: $[-5, 5]$

y-int: ± 2

R: $[-2, 2]$

Lines of Symmetry
x & y axis

Graph each equation. Identify the conic section and describe the graph and its lines of symmetry. Then find the domain and range.

$6.\ 1x^2 + 1y^2 = 49$

Both leading coefficients are 1 (same)

plus sign

Circle

D: [-7, 7]

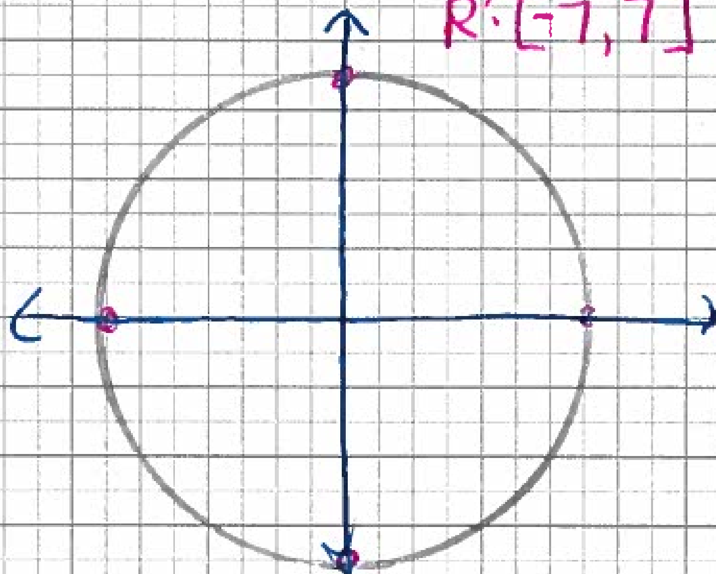
R: [-7, 7]

$x^2 = 49$

intercepts: $\boxed{x = \pm 7}$

$y^2 = 49$

$\boxed{y = \pm 7}$



Graph each equation. Identify the conic section and describe the graph and its lines of symmetry. Then find the domain and range.

$8.\ x^2 - 2y^2 = 4$

x^2 is positive major axis(x)

minus sign

$x^2 - 2y^2 = 4$

$-x^2 \qquad -x^2$

$\frac{-2y^2}{-2} = \frac{-x^2 + 4}{-2}$

$\sqrt{y^2} = \sqrt{\frac{-x^2+4}{-2}}$

$y = \pm\sqrt{\frac{4 - x^2}{-2}}$

$y_1 = \sqrt{\frac{4-x^2}{-2}}$

$y_2 = -\sqrt{\frac{4-x^2}{-2}}$

$x^2 - 2(0)^2 = 4$

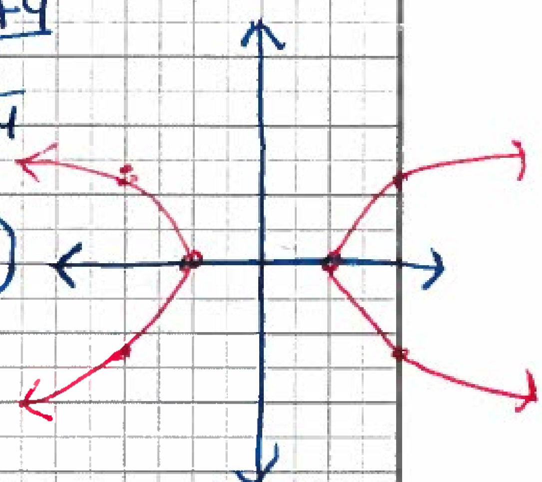
$x^2 = 4$

$x = \pm 2$

$0^2 - 2(y)^2 = 4$

$\cancel{-2y^2 = 4}$

none



x+y axes

Graph each equation. Identify the conic section and describe the graph and its lines of symmetry. Then find the domain and range.

11. $6x^2 + 24y^2 - 96 = 0$

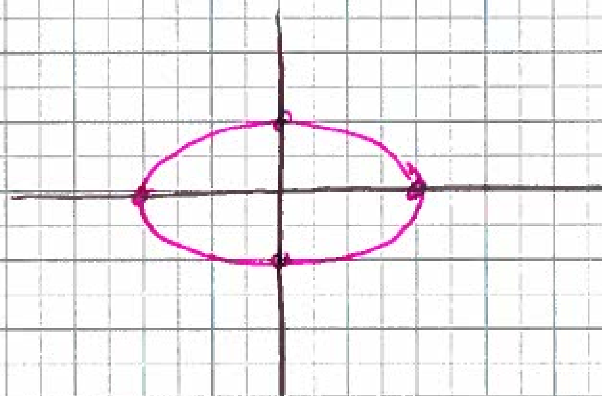
Ellipse

$6x^2 + 24y^2 = 96$

$6x^2 = 96 \quad 24y^2 = 96$

$x^2 = 16 \quad y^2 = 4$

$x = \pm 4 \quad y = \pm 2$



D: [-4,4]

R: [2,2]

Graph each equation. Identify the conic section and describe the graph and its lines of symmetry. Then find the domain and range.

12. $4x^2 + 4y^2 - 20 = 0$

Circle

Domain + Range

$[-\sqrt{5}, \sqrt{5}]$

$4x^2 + 4y^2 = 20$

$\frac{4x^2}{4} = \frac{20}{4} \quad \frac{4y^2}{4} = \frac{20}{4}$

$\sqrt{x^2} = \sqrt{5} \quad \sqrt{y^2} = \sqrt{5}$

$x = \pm\sqrt{5} \quad y = \pm\sqrt{5}$

$x = \pm 2.23 \quad y = \pm 2.23$

infinite lines of symmetry