Name ______________ Class B4 Date 2/1/18

Practice 9-3: Notes Day 17 — Rational Functions and Their Graphs

Rational Function $f(x)$ **is a function that has a** polynomial **Function in the** numerator (top) **and** denominator (bottom)

$f(x) = \frac{P(x)}{Q(x)}$ ex $f(x) = \frac{x^2-1}{x^2+x-1}$

A point of discontinuity can be an asymptote **or a** hole.

* it means the graph is NOT continuous at x=a

Vertical Asymptotes & Holes:

1st: Factor top + bottom of the fraction

2nd: set the factors on the bottom = to zero and solve for x. * If NOT Factorable → then solve by quadratic Formula

3rd: If one of the factors on the bottom caneled **with a factor on the top, that value will be a** Hole**, otherwise it will be a** vertical asymptote.

***If no value makes the** denominator zero**, then there are** no points of discontinuity.

Find any points of discontinuity for each rational function.

1. $y = \frac{\cancel{(x+3)}}{(x-4)\cancel{(x+3)}}$

$x-4=0$ $x+3=0$

$+4\ +4$

$x=4$ $x=-3$

V.A. Hole

2. $y = \frac{\cancel{(x-2)}}{x^2-4}$

$(x+2)\cancel{(x-2)}$

$x+2=0$ $x-2=0$

$-2\ -2$

V.A $\boxed{x=-2}$ $\boxed{x=2}$ Hole

3. $y = \frac{(x-3)(x+1)}{(x-2)}$

$x-2=0$

$\boxed{x=2}$

4. $y = \frac{3x(x+2)}{x(x+2)}$

$\boxed{x=0}$ $x+2=0$

$-2\ -2$

$\boxed{x=-2}$

5. $y = \frac{2}{(x+1)}$

$x+1=0$

$\boxed{x=-1}$

6. $y = \frac{4x}{x^3-9x}$

$x(x^2-9)$

$x(x+3)(x-3)$

$x=0$ $x=-3$ $x=3$

Describe the vertical asymptotes and holes for the graph of each rational function.

22. $y = \frac{\cancel{x-2}}{(x+2)\cancel{(x-2)}}$

$x+2=0$ $x-2=0$

$\boxed{x=-2 \text{ V.A.}}$ $\boxed{x=2 \text{ Hole}}$

23. $y = -\frac{\cancel{x}}{\cancel{x}(x-1)}$

$\boxed{x=0 \text{ Hole}}$ $x-1=0$ $\boxed{x=1 \text{ V.A.}}$

24. $y = \frac{5-x}{x^2-1}$

$(x+1)(x-1)$

$\boxed{x=-1,\ x=1 \text{ V.A.}}$

NO Holes - nothing canceled

25. $y = \frac{x^2-2}{x+2}$

$x+2=0$

$\boxed{x=-2}$

V.A.

26. $y = \frac{x^2-4}{x^2+4}$

sum of 2 squares

no points of discontinuity

27. $y = \frac{x+3}{x^2-9}$

28. $y = \frac{x^2-25}{x-4}$

29. $y = \frac{(x-2)(2x+3)}{(5x+4)(x-3)}$

$5x+4=0$ $x-3=0$

$-4 \ -4$ $\boxed{x=3}$

$\frac{5x}{5}=\frac{-4}{5}$

$\boxed{x=-4/5}$ $\boxed{\text{Both V.A.'s}}$

30. $y = \frac{15x^2-7x-2}{x^2-4}$

* degree = highest exponent

A rational function will have at most 1 Horizontal asymptote.

Rules for Horizontal Asymptotes:

1. If the degree of the top is greater than the bottom then there is NO horizontal asymptote.

ex $y = \frac{x^2-x-2}{x^1-2}$ top degree 2, Bottom degree 1

H.A. = NONE

Leading coeffient is the # in front of the highest degree

2. If the degree of the top = bottom $y = \dfrac{\text{leading coefficient of top}}{\text{leading coefficient of bottom}}$

ex $y = \dfrac{2x^2-1}{5x^2+6}$ (degree 2) (degree 2) $\boxed{\text{H.A.: } y = \frac{2}{5}}$

3. If the degree of the top is less than the bottom

then H.A: $y=0$ ex $y = \dfrac{x}{x^2+1}$ (degree 1) (degree 2)

Find the horizontal asymptote of the graph of each rational function.

7. $y = \dfrac{2}{x-6}$ degree 0 degree 1 $0<1$ $\boxed{\text{H.A.: } y=0}$

8. $y = \dfrac{1x+2}{1x-4}$ degree 1 degree 1 $y = \frac{1}{1}$ $\boxed{y=1}$

9. $y = \dfrac{(x+3)}{2(x+4)}$ $\dfrac{1x^1+3}{2x^1+8}$ deg.1 deg1 $\boxed{y=\frac{1}{2}}$

10. $y = \dfrac{2x^2+3}{x^2-6}$ degree 2 degree 2 $y = \frac{2}{1}$ $\boxed{y=2}$

11. $y = \dfrac{3x-12}{x^2-2}$ deg.1 deg 2 $1<2$ $\boxed{y=0}$

12. $y = \dfrac{3x^3-4x+2}{2x^3+3}$ $\boxed{y=\frac{3}{2}}$

Sketch the graph of each rational function.

13. $y = \dfrac{3}{x-2}$

14. $y = \dfrac{3}{(x-2)(x+2)}$ deg 0 deg 2 x^2 $\boxed{y=0}$

15. $y = \dfrac{x}{x(x-6)}$

16. $y = \dfrac{2x}{x-6}$

17. $y = \dfrac{x^2-1}{x^2-4}$

18. $y = \dfrac{2x^2+10x+12}{x^2-9}$

19. $y = \dfrac{x}{x^2+4}$

20. $y = \dfrac{x+2}{x-1}$

21. $y = \dfrac{x+3}{x+1}$