8-2 Properties of Exponential Functions

Day 3

What You'll Learn

- To identify the role of constants in $y = ab^{cx}$
- To use e as a base

. . . And Why

To model the half-life of a radioactive substance, as in Example 3

$y = a|x-h|+k$

Check Skills You'll Need GO for Help Lessons 2-6, 5-3, and 7-4

Write an equation for each translation.

1. $y = |x|$, 1 unit up, 2 units left
2. $y = -|x|$, 2 units down
3. $y = x^2$, 2 units down, 1 unit right
4. $y = -x^2$, 3 units up, 1 unit left

Write each equation in simplest form. Assume that all variables are positive.

5. $y = \left(x^{-\frac{5}{4}}\right)^4$
6. $y = \left(x^{-\frac{1}{7}}\right)^{-7}$
7. $y = \left(x^{\frac{5}{6}}\right)^6$

8. Use the formula for simple interest $I = Prt$. Find the interest for a principal of \$550 at a rate of 3% for 2 years.

Dec 15-8:36 AM

Parent Functions

Linear: $y = x$

Absolute Value: $y = |x|$

Quadratic: $y = x^2$

Cubic: $y = x^3$

Square Root: $y = \sqrt{x}$

Exponential: $y = b^x$, $a = 1$ or $y = ab^x$

$y = b^x + k$ move up, asymptote $y = k$

$y = b^x - k$ move down, asymptote

$y = b^{x+h}$ move left h

$y = b^{x-h}$ move right h

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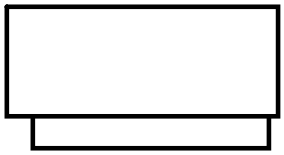
if "a" is negative there is a reflection

$y = 2^{-x}$ reflects over y-axis

$y = -2^{x}$ reflects over x-axis



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h, K form for an exponential function

stretch or shrink

moves left or right

$$y = ab^{x-h} + k$$

a = original amount or initial value

b is the growth or decay factor

shifts up or down

Dec 15-9:08 AM

Describe any shifts and/or reflections

1. $y = 3^{x-6}$ right 6 $x-6=0$, $+6\ +6$, $x=6$
2. $y = 3^{x+6}$ Left 6
3. $y = 3^{x}+6$ up 6
4. $y = 3^{x}-6$ down 6

all are growth $b=3$

5. $y = 3^{-x}$ reflection/y axis
6. $y = -3^{x}$ reflection/x-axis

Dec 15-9:09 AM

1 Comparing Graphs

The function $f(x) = b^x$ is the parent of a family of exponential functions for each value of b. The factor a in $y = ab^x$ stretches, shrinks, and/or reflects the parent.

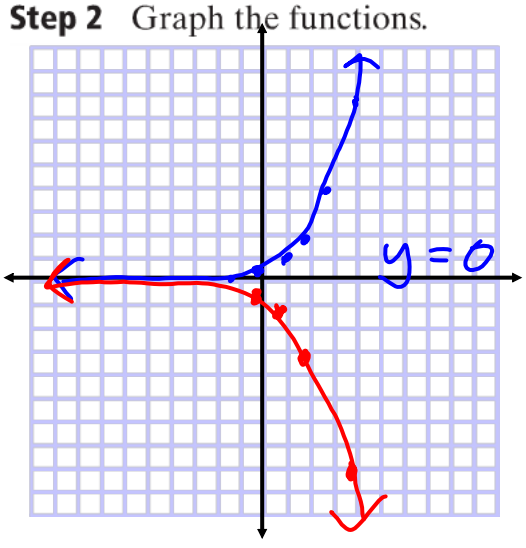
1 EXAMPLE **Graphing $y = ab^x$ for $0 < |a| < 1$**

reflect over x-axis

Graph $y = \frac{1}{2} \cdot 2^x$ and $y = -\frac{1}{2} \cdot 2^x$. Label the asymptote of each graph.

x	$y = \frac{1}{2} \cdot 2^x$	$y = -\frac{1}{2} \cdot 2^x$
-2	.125	-.125
-1	.25	-.25
0	.5	-.5
1	1	-1
2	2	-2
3	4	-4
4	8	-8
2		
3		

Step 2 Graph the functions.



Dec 15-8:46 AM

A horizontal shift $y = ab^{x-h}$ is the same as the vertical stretch or shrink $y = (ab^{-h})b^x$. A vertical shift $y = ab^x + k$ also shifts the horizontal asymptote from $y = 0$ to $y = k$.

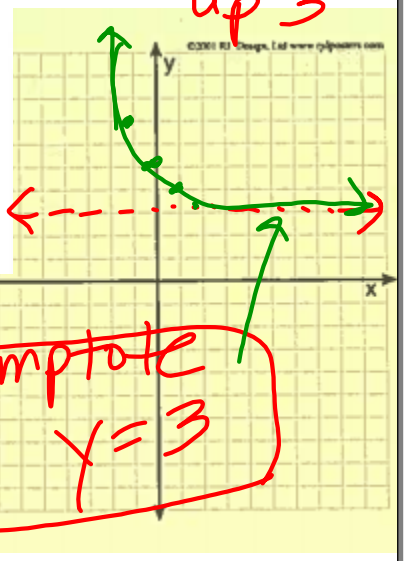
2 EXAMPLE Translating $y = ab^x$

Graph the stretch $y = 8\left(\frac{1}{2}\right)^x$ and then the translation $y = 8\left(\frac{1}{2}\right)^{x+2} + 3$.

Step 1 Graph $y = 8\left(\frac{1}{2}\right)^x$. The horizontal asymptote is $y = 0$.

Step 2 For $y = 8\left(\frac{1}{2}\right)^{x+2} + 3$, $h = -2$ and $k = 3$. So shift the $y = 8\left(\frac{1}{2}\right)^x$ graph 2 units left and 3 units up. The horizontal asymptote is $y = 3$.

-2	11
-1	7
0	5
1	4
2	3.5
3	3.25
4	3.125

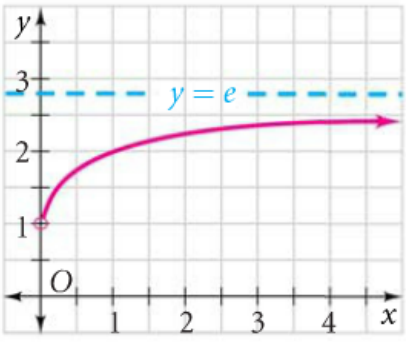


Dec 15-8:51 AM

4 EXAMPLE Evaluating e^x

At the right is part of the graph of the function $y = \left(1 + \frac{1}{x}\right)^x$. One of the graph's asymptotes is $y = e$, where e is an irrational number approximately equal to 2.71828.

Exponential functions with a base of e are useful for describing continuous growth or decay. Your graphing calculator has a key for e^x.



4 Use the graph of $y = e^x$ to evaluate each expression to four decimal places.

a. e^4 **b.** e^{-3} **c.** $e^{\frac{1}{2}}$

Dec 15-9:01 AM

Definition Continuously Compounded Interest Formula

$A = Pe^{rt}$

A: amount in account; P: principal; r: rate of interest (annual); t: time in years

P ↘ r = .043

5 Suppose you invest $1300 at an annual interest rate of 4.3% compounded continuously. Find the amount you will have in the account after three years.

$A = 1300e^{(.043*3)}$ ↑ t

30. Savings A student wants to save $8000 (A) for college in five years. How much should be put into an account that earns 5.2% annual interest compounded continuously?

$\frac{8000}{e^{(.052*5)}} = \frac{P \cdot e^{(.052*5)}}{e^{(.052*5)}}$

37. Investment How long would it take to double your principal at an annual interest rate of 8% compounded continuously?

Dec 15-9:19 AM

Some exponential functions are of the form $y = ab^{cx}$, where c is a nonzero constant.

3 EXAMPLE Real-World Connection

Medicine The half-life of a radioactive substance is the time it takes for half of the material to decay. A hospital prepares a 100-mg supply of technetium-99m, which has a half-life of 6 hours. Make a table showing the amount of technetium-99m that remains at the end of each 6-hour interval for 36 hours. Then write an exponential function to find the amount of technetium-99m that remains after 75 hours.

The amount of technetium-99m is reduced by one half each 6 hours.

Number of 6-h Intervals	0	1	2	3	4	5	6
Number of Hours Elapsed	0	6	12	18	24	30	36
Technetium-99m (mg)	100	50	25	12.5	6.25	3.13	1.56

Relate The amount of technetium-99m is an exponential function of the number of half-lives. The initial amount is 100 mg. The decay factor is $\frac{1}{2}$. One half-life equals 6 h.

Define

Write $y =$

Dec 15-9:01 AM

15. Botany Phosphorus-32 is used to study a plant's use of fertilizer. It has a half-life of 14.3 days. Write the exponential decay function for a 50-mg sample. Find the amount of phosporus-32 remaining after 84 days.

16. Public Works Iodine-131 is used to find leaks in water pipes. It has a half-life of 8.14 days. Write the exponential decay function for a 200-mg sample. Find the amount of iodine-131 remaining after 72 days.

Dec 15-9:22 AM

HW: 8.2 wkst
(1-6, 10-12, 15, 16, 25, 26)

Dec 15-9:22 AM