

**Practice 11-5: Notes**

Day 34

Geometric Series

If a geometric series is finite,  
the sum formula is

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

r = common ratio

1st Find r

$$r = \frac{a_2}{a_1} \quad r = \frac{a_3}{a_2} \quad r = \frac{a_4}{a_3}$$

Evaluate the finite series for the specified number of terms.

19.  $40 + 20 + 10 + \dots; n = 10$

$r = \frac{20}{40} = .5$   
 $a_1 = 40$   
 $S = \frac{40(1-.5^{10})}{1-.5}$   
 $S \approx 79.921875$

20.  $4 + 12 + 36 + \dots; n = 15$

$r = 3$   
 $S = \frac{4(1-3^{15})}{1-3}$   
 $S = 28,697,812$

21.  $15 + 12 + 9.6 + \dots; n = 40$

$r = .8$

22.  $27 + 9 + 3 + \dots; n = 100$

$r = \frac{1}{3}$   
 $S = \frac{27(1-(\frac{1}{3})^{100})}{1-(\frac{1}{3})}$   
 $S = 40.5$

23.  $0.2 + 0.02 + 0.002 + \dots; n = 8$

$r = .1$

24.  $100 + 200 + 400 + \dots; n = 6$

$r = 2$

arithmetic explicit  $a_n = a_1 + d(n-1)$

geometric finite sum

$$\frac{a_1(1-r^n)}{1-r}$$

Determine whether each series is arithmetic or geometric. Then evaluate the series to the given term.

15.  $2 + 5 + 8 + 11 + \dots; S_9$

$A, d = 3$   
 $a_9 = 2 + 3(9-1)$   
 $a_9 = 26$

$\frac{9}{2}(2+26)$   
 $= 126$

$$\frac{n}{2}(a_1 + a_n)$$

arithmetic  $G, r = .5$   
 finite sum

16.  $\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \dots; S_8$

$\frac{\frac{1}{8}(1-.5^8)}{1-.5} = \frac{255}{1024}$   
 or  $\approx .2490$

17.  $-3 + 6 - 12 + 24 - \dots; S_{10}$

$G, r = -2$

18.  $-2 + 2 + 6 + 10 + \dots; S_{12}$

$A, d = 4$

$a_{12} = -2 + 4(12-1) = 42$

Sum =  $\frac{12}{2}(-2 + 42) = 240$

If a geometric series is infinite, you must decide if it converges or diverges.

Always find the common ratio first.

If the  $|r| < 1$ , then the series converges (which means it gets closer and closer to the sum, S).

And then you can use the formula to find the sum of the infinite geometric series:

$$S = \frac{a_1}{1-r}$$

If  $|r| \geq 1$ , then there is NO Sum and the series diverges.  
Or approaches NO Limit.

ex)  $4 + 8 + 16 + \dots$

$r = 2$

$|2| \geq 1$   
 $2 \geq 1$

NO  
Sum, diverges

Decide whether each infinite geometric series diverges or converges. State whether each series has a sum.

1.  $3 + \frac{3}{2} + \frac{3}{4} + \dots$

2.  $4 + 2 + 1 + \dots$

3.  $17 + 15.3 + 13.77 + \dots$

NO Sum  $\rightarrow |r| \geq 1$   $|r| < 1 \leftarrow S = \frac{a_1}{1-r}$

$r = 0.9 < 1$   
converges  
yes

$\frac{17}{1-0.9}$   
170

$\frac{-20}{1-0.4}$   
 $\downarrow$  -33.3

4.  $6 + 11.4 + 21.66 + \dots$

5.  $-20 - 8 - 3.2 - \dots$

6.  $50 + 70 + 98 + \dots$

$r = 1.9 \geq 1$   
diverges  
no

$r = 0.4 < 1$   
converges  
yes

$r = 1.4 \geq 1$   
diverges  
no

Evaluate each infinite series that has a sum.

7.  $\sum_{n=1}^{\infty} 5 \left(\frac{2}{3}\right)^{n-1}$   
 $a_1 \cdot r^{n-1}$   
 $a_1 = 5$   
 $\frac{2}{3} < 1$   
 $\frac{5}{1-\frac{2}{3}} = 15$

8.  $\sum_{n=1}^{\infty} (-2.1)^{n-1}$   
 $r = -2.1 > 1$   
NO SUM

9.  $\sum_{n=1}^{\infty} \left(-\frac{1}{2}\right)^{n-1}$   
 $a_1 = \left(-\frac{1}{2}\right)^{1-1} = 1$   
 $S = \frac{1}{1-\frac{1}{2}} = \frac{2}{1}$

10.  $\sum_{n=1}^{\infty} 2 \left(\frac{5}{3}\right)^{n-1}$   
 $\frac{5}{3} > 1$   
NO SUM

**Evaluate each infinite geometric series.**

11.  $8 + 4 + 2 + 1 + \dots$

12.  $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$

13.  $120 + 96 + 76.8 + 61.44 + \dots$

14.  $1000 + 750 + 562.5 + 421.875 + \dots$

25. This month, Julia deposits \$400 to save for a vacation. She plans to deposit 10% more each successive month for the next 11 months. How much will she have saved after the 12 deposits?
26. Suppose your business made a profit of \$5500 the first year. If the profit increases 20% per year, find the total profit over the first 5 yr.
27. The end of a pendulum travels 50 cm on its first swing. Each swing after the first travels 99% as far as the preceding one. How far will the pendulum travel before it stops?
28. A seashell has chambers that are each 0.82 times the length of the next chamber. The outer chamber is 32 mm around. Find the total length of the shell's spiraled chambers.
29. The first year a toy manufacturer introduces a new toy, its sales total \$495,000. The company expects its sales to drop 10% each succeeding year. Find the total expected sales in the first 6 yr. Find the total expected sales if the company offers the toy for sale for as long as anyone buys it.

