11-6 Area Under a Curve

Check Skills You'll Need

GO for Help Skills Handbook page

Find the area of a rectangle with the given length and width.

1. $\ell = 4$ ft, $w = 1$ ft
2. $\ell = 5.5$ m, $w = 0.5$ m
3. $\ell = 6.2$ cm, $w = 0.1$ cm
4. $\ell = 9\frac{1}{2}$ in., $w = 3\frac{5}{8}$ in.

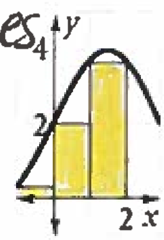
New Vocabulary • inscribed rectangles • circumscribed rectangles

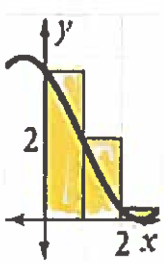
Feb 28-1:21 PM

11-6 1 Finding Area Under a Curve

You can easily calculate the exact area under part of a line parallel to the x-axis, but it is not so easy to calculate the exact area under part of a curve. You can use rectangles to estimate the area under a curve and analyze data.

Inscribed Rectangles are completely under the curve. The approximation is Less than the area.





Circumscribed Rectangles are partially above the curve. The approximation is greater than the area.

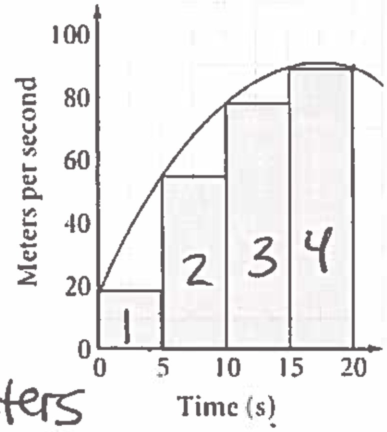
Apr 11-7:43 AM

1 EXAMPLE **Real-World Connection**

Data Analysis The curve at the right approximates the speed of a peregrine falcon during the first 20 s of a high-speed dive.

Real-World Connection

Adult peregrine falcons can reach speeds of 200 mi/h in a dive.



a. What does the area under the curve represent?

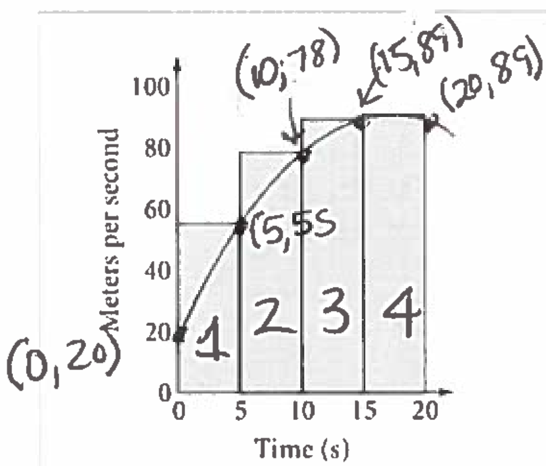
$\text{Area} = \frac{\text{meters}}{\cancel{\text{second}}} \cdot \cancel{\text{seconds}} = \text{meters}$

Answer: Total distance traveled by the falcon

b. Use inscribed rectangles 5 units wide to estimate the area under the curve.

$\text{Area} = \underset{1}{5(18)} + \underset{2}{5(55)} + \underset{3}{5(78)} + \underset{4}{5(89)}$

~~Inscribed Area = 1200 meters~~



Average of Inscribed & Circumscribed - should be close to the actual

$\frac{1200 + 1565}{2} = 1382.5$

$\underset{1}{5(55)} + \underset{2}{5(78)} + \underset{3}{5(89)} + \underset{4}{5(91)}$

Circumscribed Area: 1565 meters

Summation notation can be used to represent the area of a series of rectangles to approximate the area under the curve.

$$A = \sum_{n=1}^{b \leftarrow \text{\# of rectangles}} \underset{\text{width}}{(w)}\, f(a_n)$$

$f(a_n)$ ← Function value at a_n

ex

$f(x) = -x^2 + 5,\ 0 \le x \le 2$ ← DOMAIN

$$A = \sum_{n=1}^{4 \leftarrow 4 \text{ rectangles}} (0.5)\, f(a_n)$$

↑ rectangles are .5 wide

Apr 11-7:12 AM

~~p. 638 15 y = 4 − (1/4)x²~~

Average = 7.25

Calculator = $7.\overline{3}$

A = 6.25

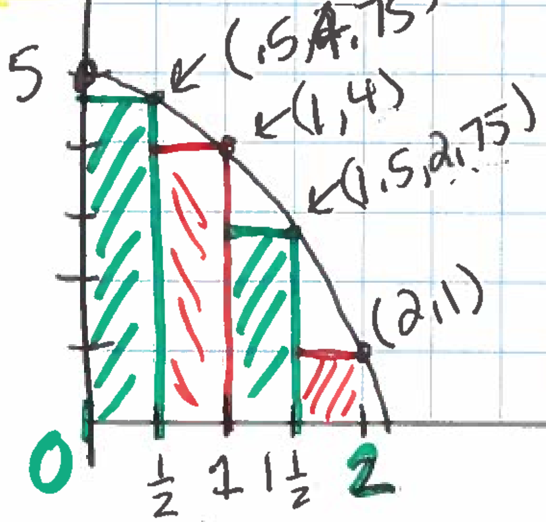
Inscribed

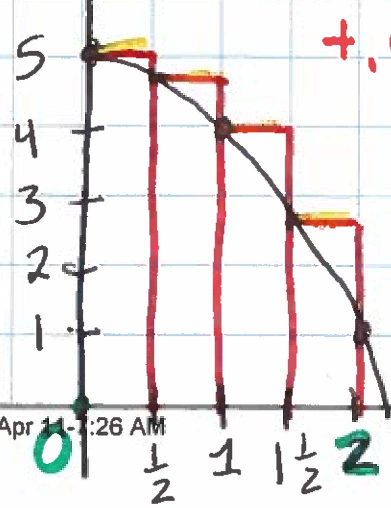
$.5(4.75) + .5(4) + .5(2.75) + .5(1)$

Circumscribed

$A = .5(5) + .5(4.75) + .5(4) + .5(2.75)$

$A = 8.25$





You can use a graphing calculator to find the exact area under a curve.

3 EXAMPLE Using a Graphing Calculator

Graph the function $f(x) = -2x^2 + 5$. Find the area under the curve for the domain $-1 \le x \le 1.5$.

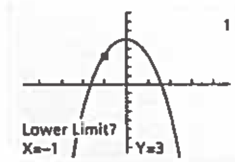
Step 1 Input the equation. Adjust the window values. Y= GRAPH

Step 2 Access the $\int f(x)dx$ feature from the CALC menu.

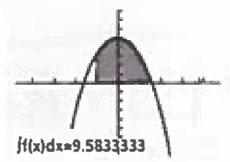
2nd trace #7:

Step 3 Use the lower limit of $x = -1$.

Step 4 Use the upper limit of $x = 1.5$.



Xmin=-4.7 Ymin=-7
Xmax=4.7 Ymax=8
Xscl=1 Yscl=1



$y = -2x^2 + 5$

The area under the curve between $x = -1$ and $x = 1.5$ is about 9.583 units^2.

3 Use the equation from Example 3 and a graphing calculator. Find the area under the curve for each domain.

a. $0 \le x \le 1$ 4.3 b. $-1 \le x \le 1$ 8.6 c. $-1.5 \le x \le 0$ 5.25

Lower Limit upper limit

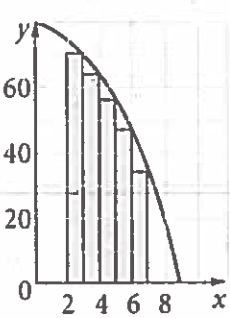
Apr 11-7:49 AM

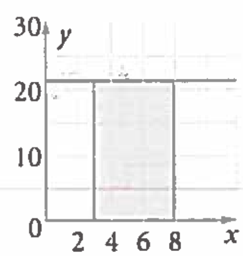
Multiply y·x

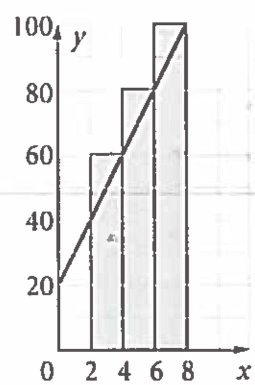
Given each set of axes, what does the area under the curve represent?

1. y-axis: production rate, x-axis: time items produced / HR(time) · HR(time)
2. y-axis: rate of growth, x-axis: time total items produced
3. y-axis: miles per gallon, x-axis: gallons
4. y-axis: distance traveled per year, x-axis: years
5. y-axis: price per pound of gold, x-axis: pounds of gold

Use the given rectangles to estimate area under the curve.

6. 

7. 

8. 

Change in agenda!

Feb 28-1:26 PM

HW: p. 637-638 (1-10, 18-23)