

11.4 Arithmetic Series

p. 619 ✓ Check your skills Needed
Find each sum.

$$1.) \quad 2 + 3.5 + 5 + 6.5 + 8 = 25$$

$$2.) \quad -17 + (-13) + (-9) + (-5) + (-1) + 3 = -42$$

Write the explicit formula for the sequence..

$$3.) \quad 4, 6, 8, 10, 12, \dots \quad \text{adding } 2 \rightarrow d = 2$$

$$a_n = a_1 + (n-1)d \rightarrow a_n = 4 + (n-1)2 \\ = 4 + 2n - 2$$

$$\dots \quad \boxed{a_n = 2n + 2}$$

$$5.) \quad -17, -23, -29, -35, \dots \quad d = -6$$

$$a_n = -17 + (n-1)(-6) \\ = -17 + -6n + 6$$

$$\boxed{a_n = -6n - 11}$$

p. 619 Activity: Arithmetic Series

1, 2, 3, 4, 5, ..., 96, 97, 98, 99, 100

1. Arithmetic

2. 101

3. $101(50) = 5050$

4. $5 + 10 + 15 + 20 \dots + 50$

$$5 + 50 = 55 \\ 10 + 45 = 55$$

$$55(5) = \boxed{275}$$

Sum of a Finite Arithmetic Series

$$S_n = \frac{n}{2} (a_1 + a_n) \quad n = \# \text{ OF TERMS}$$

↑
↑
 1st term last term

Finite sequence

6, 9, 12, 15, 18

Finite series

6 + 9 + 12 + 15 + 18

Infinite sequence

3, 7, 11, 15, ...

Infinite Series

3 + 7 + 11 + 15 + ...

A series is the sum of the terms in the sequence.
(has plus signs)

p. 620 Write the related series for each finite sequence. Then evaluate the series.

b) 100, 125, 150, 175, 200, 225 ← sequence

$\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 100 & 125 & 150 & 175 & 200 & 225 \end{matrix}$ ← series
 $a_1 \qquad \qquad \qquad a_n$

n = 6
6 terms

$$S_n = \frac{6}{2} (100 + 225) = 975$$

Limits are the least and greatest integral values of n .

Σ ← "sigma" is a summation symbol

$$\sum_{n=1}^3 (5n+1)$$

3 ← upper limit
n=1 ← lowest limit (least value of n)
(5n+1) ← explicit formula

$$\underline{5(1)+1} + \underline{5(2)+1} + \underline{5(3)+1}$$

$$6 + 11 + 16 = \textcircled{33}$$

✓ #3 Use summation notation to write each series for the specified # of terms.

a) $1 + 2 + 3 + \dots$; $n = 6$

$$\sum_{n=1}^6 n$$

Find the explicit formula. $d = 1$

$$a_n = 1 + (n-1)(1)$$

$$= 1 + n - 1$$

$$a_n = n$$

$a_1 \ a_2 \ a_3 \ a_4 \ \dots \ a_9$

b) $3 + 8 + 13 + 18 + \dots$; $n = 9$

$$\sum_{n=1}^9 (5n-2)$$

$$a_n = a_1 + (n-1)d \quad d = 5$$

$$a_n = 3 + (n-1)5$$

$$= 3 + 5n - 5$$

$$= 5n - 2$$

→ explicit arithmetic formula

✓ #4

$$\sum_{n=1}^{10} (n-3)$$

Find the 1st
+ last
terms

$$\frac{n}{2}(a_1 + a_n)$$

$$a_1 = 1 - 3 = -2$$

$$a_{10} = 10 - 3 = 7$$

$$\frac{10}{2}(-2 + 7)$$

$$= 5(5)$$

$$= \textcircled{25}$$

b) $\sum_{n=1}^4 \left(\frac{1}{2}n + 1\right)$

$$\frac{n}{2}(a_1 + a_n)$$

$$a_1 = \frac{1}{2}(1) + 1 = 1.5$$

$$a_4 = \frac{1}{2}(4) + 1 = 3$$

$$\frac{4}{2}(1.5 + 3)$$

$$= \boxed{9}$$

c) $\sum_{n=2}^5 n^2$

$$\begin{array}{cccc} \swarrow a_2 & \swarrow a_3 & \swarrow a_4 & \swarrow a_5 \\ 4 & + & 9 & + & 16 & + & 25 \end{array}$$

1st $\rightarrow a_2 = 2^2 = 4$

last $\rightarrow a_5 = 5^2 = 25$

of terms = 4

$$\frac{n}{2}(a_1 + a_n)$$

$$\frac{4}{2}(4 + 25)$$

$$= 2(29)$$

$$= \textcircled{58}$$