WARM-UPS p.445(1 - 4, 6, 10)

Checkpoint Quiz 1 Lessons 8-1 through 8-2

Identify each function as modeling either exponential growth or exponential decay. What percent of increase or decrease does the function model?

1. $y = 15(1.45)^x$ $1+R=1.45$
2. $y = 0.32(0.99)^x$ $1-R=.99$ $R=.01(100)=1\%$
3. $y = 0.1(1.7)^x$
4. $y = 7.3(0.8)^x$

Graph each function.

5. $y = 3^x$
6. $y = 2^x + 1$
7. $y = (0.25)^x$
8. $y = 4^x - 5$

9. **Open-Ended** Describe a real-world problem that you could model with an exponential growth function.

10. **Chemistry** An element has a half-life of 30 hours. Write the exponential decay function for a 100-mg sample. Use the function to find the amount of the element remaining after 100 hours.

$\frac{1}{2}$ = decay factor

$y = 100\left(\frac{1}{2}\right)^{\frac{1}{30}x}$

$y = 100\left(\frac{1}{2}\right)^{\frac{1}{30}\cdot 100}$

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8-3

Multiple Representations

Logarithmic Functions as Inverses

Day 6

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What You'll Learn

- To write and evaluate logarithmic expressions
- To graph logarithmic functions

. . . And Why

To compare the acidities of milk and lemon juice, as in Example 4

Check Skills You'll Need **GO for Help** Lessons 6-4 and 7-7

Solve each equation.

1. $8 = x^3$ $x \cdot x \cdot x$ $8 = 2 \cdot 2 \cdot 2$ $x = 2$
2. $x^{\frac{1}{4}} = 2$ $(x^{\frac{1}{4}})^4 = 2^4$ $x = 16$
3. $27 = 3^x$ $27 = 3^3$ $x = 3$
4. $4^6 = 4^{3x}$ $x = 2$

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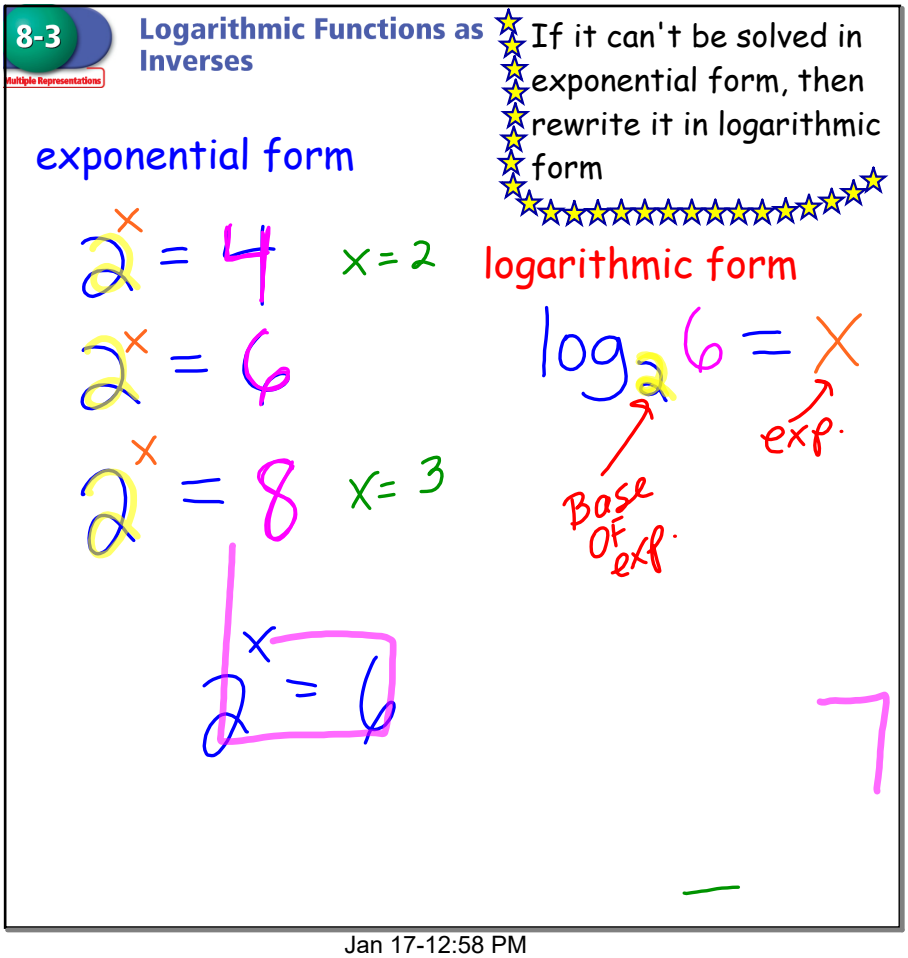

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On the Graphing Calculator...

$\log_2 6$

Base

older calc: $\log(6) \div \log(2)$

newer calc: alpha window 5

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8-3 Logarithmic Functions as Inverses

Multiple Representations

Read $\log_b y$ as "log base b of y."

Definition **Logarithm**

The **logarithm** to the base b of a positive number y is defined as follows:

If $y = b^x$, then $\log_b y = x$.

☆ The exponent x in the exponential expression b^x is the logarithm in the equatio $\log_b y = x$. The base b in b^x is the same as the base b in the logarithm. In both cases, $b \neq 1$ and $b > 0$.

☆ A positive number b raised to any power x cannot equal a number y less than or equal to zero. Therefore, the logarithm of a negative number or zero is undefined.

log(-2) = undefined

2 EXAMPLE **Writing in Logarithmic Form**

Write $25 = 5^2$ in logarithmic form.

log(0) = undefined

If $y = b^x$, then $\log_b y = x$. **Write the definition.**

If $25 = 5^2$, then $\log_5 25 = 2$. **Substitute.**

The logarithmic form of $25 = 5^2$ is $\log_5 25 = 2$.

2 Write each equation in logarithmic form.

a. $729 = 3^6$ **b.** $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ **c.** $10^0 = 1$

$\log_3 729 = 6$

$\log_{\frac{1}{2}} \left(\frac{1}{8}\right) = 3$

$\log_{10} 1 = 0$

can't = 0 or be neg.

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8-3 Logarithmic Functions as Inverses

Multiple Representations

Evaluating Logarithms

Evaluate $\log_8 16$.

$\log_8 16 = x$

$16 = 8^x$

Write an equation in logarithmic form.

Convert to exponential form.

Write each side using base 2. (same base)

Power Property of Exponents

Set the exponents equal to each other.

Solve for x.

$8^x = 16$

$2^{3x} = 2^4$

$\frac{3x}{3} = \frac{4}{3}$

$x = \frac{4}{3}$

$8 = 2^3$ (4, 2, 2, 2)

$16 = 2^4$ (4, 4, 2, 2, 2, 2)

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3 Evaluate each logarithm.

a. $\log_{64} \frac{1}{32} = x$

$64^x = \frac{1}{32}$

$2^{6x} = \frac{1}{2^5}$

$2^{6x} = 2^{-5}$

$\frac{6x}{6} = \frac{-5}{6}$

$x = \frac{-5}{6}$

c. $\log_{10} 100 = 2$

$\log_{10} 100 = x$

$10^x = 100$

$10^x = 10^2$

b. $\log_9 27$

$\log_9 27 = x$

$9^x = 27$

$(3^2)^x = 3^3$

$3^{2x} = 3^3$

Set exponents =

$\frac{2x}{2} = \frac{3}{2}$

$x = 1.5$

$9 = 3^2$

$27 = 3^3$

$3 \cdot 3 \cdot 3$

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A **common logarithm** is a logarithm that uses base 10. You can write the common logarithm $\log_{10} y$ as $\log y$.

Scientists use common logarithms to measure acidity, which increases as the concentration of hydrogen ions in a substance increases. The pH of a substance equals $-\log[H^+]$, where $[H^+]$ is the concentration of hydrogen ions.

4 EXAMPLE **Real-World Connection**

Chemistry The pH of lemon juice is 2.3, while the pH of milk is 6.6. Find the concentration of hydrogen ions in each substance. Which substance is more acidic?

Lemon juice	Milk
$pH = -\log[H^+]$	$pH = -\log[H^+]$
$2.3 = -\log[H^+]$	$6.6 = -\log[H^+]$
$\log[H^+] = -2.3$	$\log[H^+] = -6.6$
$[H^+] = 10^{-2.3}$	$[H^+] = 10^{-6.6}$
$\approx 5.0 \times 10^{-3}$	$\approx 2.5 \times 10^{-7}$

The $[H^+]$ of lemon juice is about 5.0×10^{-3}. The $[H^+]$ of milk is about 2.5×10^{-7}. Lemon juice has a higher concentration of hydrogen ions, so it is more acidic.

4 Find the concentration of hydrogen ions in seawater of pH 8.5.

$10^{-8.5} = 3.16\text{E-}9$

3.16×10^{-9}

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Evaluate Each Logarithm.

1. $\log_2 32$
2. $\log_{10}(-100)$
3. $\log_5 \frac{1}{25}$

Write each eq. in exponential form

4. $\log 0.01 = -2$
5. $\log_3 27 = 3$

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Assignment:

p.450 (6 - 34 Even, 43 - 46 all, 53 - 61 odd)

Write each equation in logarithmic form.

6. $49 = 7^2$ **7.** $10^3 = 1000$ **8.** $625 = 5^4$ **9.** $\frac{1}{10} = 10^{-1}$

10. $8^2 = 64$ **11.** $4 = \left(\frac{1}{2}\right)^{-2}$ **12.** $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$ **13.** $10^{-2} = 0.01$

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Evaluate each logarithm.

14. $\log_2 16$ **15.** $\log_4 2$ **16.** $\log_8 8$ **17.** $\log_4 8$

18. $\log_2 8$ **19.** $\log_{49} 7$ **20.** $\log_5 (-25)$ **21.** $\log_3 9$

22. $\log_2 2^5$ **23.** $\log_{\frac{1}{2}} \frac{1}{2}$ **24.** $\log 10{,}000$ **25.** $\log_5 125$

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The pH of each food is given. Find the concentration of hydrogen ions $[H^+]$.

26. maple syrup, 5.2 **27.** lime juice, 2.2 **28.** egg white, 8.0

29. cider vinegar, 3.1 **30.** condensed milk, 6.3 **31.** soy sauce, 4.9

32. tomato juice, 4.0 **33.** watermelon, 5.4 **34.** mustard, 3.6

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Use your calculator to evaluate each logarithm to four decimal places. Then find the largest integer that is less than the value of the logarithm.

41. $\log 5$ **42.** $\log (6.1 \times 10^{-5})$ **43.** $\log 0.08$ **44.** $\log 200$

45. $\log \frac{1}{6}$ **46.** $\log 17.52$ **47.** $\log (1.3 \times 10^{7})$ **48.** $\log \frac{13}{4}$

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Write each equation in exponential form.

53. $\log_2 128 = 7$ **54.** $\log 0.0001 = -4$ **55.** $\log_7 16{,}807 = 5$

56. $\log_6 6 = 1$ **57.** $\log_4 1 = 0$ **58.** $\log_3 \frac{1}{9} = -2$

59. $\log_2 \frac{1}{2} = -1$ **60.** $\log 10 = 1$ **61.** $\log_2 8192 = 13$

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