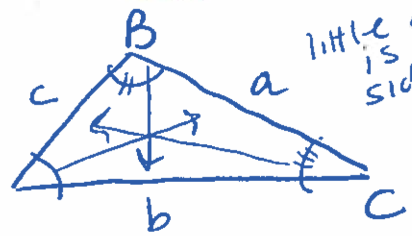
14.4 Area & The Law of Sines

If you have a right △, or a triangle where you know the base and height, you can use the formula $A = \frac{1}{2}bh$ to find the area of the △.

When you have an oblique △, you can find the area if you know 2 sides and the angle between them. Think SAS (side angle side) back from Geometry.



little a is a side

capital letters for angles

The Formula Is

$A = \frac{1}{2} b \cdot c \cdot \sin A$

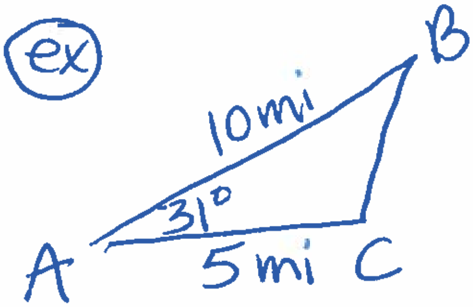
OR $A = \frac{1}{2} a \cdot c \cdot \sin B$

OR $A = \frac{1}{2} \cdot a \cdot b \cdot \sin C$

OR think

$\boxed{A = \frac{1}{2}(\text{side})(\text{side})\sin(\text{angle between the sides})}$

ex



$A = \frac{1}{2}(10)(5)\sin 31°$

mode on calculator must be in degrees

$\boxed{A \approx 12.9 \text{ mi}^2}$

ex A △ has side lengths 12in and 15in. The angle in between them is 24°. Find the area of the △.

$A = \frac{1}{2}(12)(15)\sin 24°$

$\boxed{A = 36.6 \text{ in}^2}$

Combining those 3 equations

$\frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$ (by the transitive property)

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

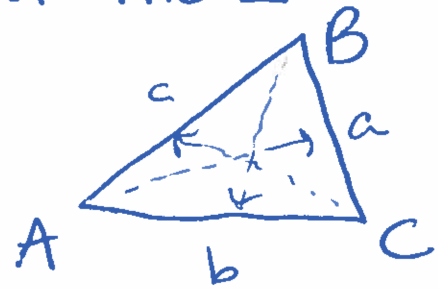
$\div$ out $\frac{1}{2}abc$ out of each and you get

The Law of Sines

when we did right Δ trig (sin, cos, tan) they only worked for right Δ's.

The Law of Sines works for all Δ's (right, acute or obtuse)

This relationship is always between an $\angle$ of the Δ and the side across from it.



you can use Law of Sines if you know the measures of

1) 2 $\angle$'s and any side <u>OR</u>

2) 2 sides and the $\angle$ opposite one of them.

* 180° in every Δ

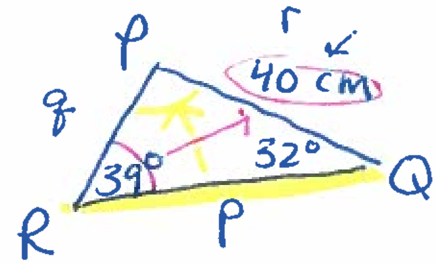
* If you know 2 $\angle$'s you can always find the 3rd.

(Ex) 2 p. 802

* **Finding a side**

In $\triangle PQR$ $m\angle R = 39°$, $m\angle Q = 32°$ and $PQ = 40$ cm.

Find RQ.



1st: Draw and label the picture. (Doesn't have to be to scale)

2nd: since you are finding RQ(p), you need to find the ∠ across from it (angle P).

$\angle P = 180° - 39° - 32°$

$\angle P = 109°$.

3rd: set up a proportion.

4th: cross multiply + ÷

$$\frac{p}{\sin P} = \frac{r}{\sin R}$$

$$\frac{p}{\sin 109°} = \frac{40}{\sin 39°}$$

$$\frac{\cancel{\sin(39°)} \cdot p}{\cancel{\sin(39)}} = \frac{40(\sin(109))}{\sin(39)}$$

$$\overline{RQ} = p = 60.1\text{cm}$$

(ex) Try Quick ✓ # 2 on p. 802 on your own

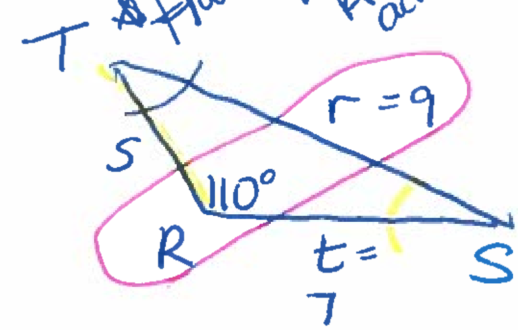
(ex) 3 **Finding an angle**

p. 802

In $\triangle RST$, $t = 7$, $r = 9$, $m\angle R = 110°$.

Find $m\angle S$.

*Have to find ∠T 1st since we know the side across from it



1st: Draw pic.

∠R must be 110° which doesn't matter which ∠ is T or S but little t and little s must be across from those angles

2nd: set up proportion

$$\frac{9}{\sin 110°} = \frac{7}{\sin T}$$

* cross multiply and ÷ then take inverse sine of answer

$T = \sin^{-1}(7 * \sin 110° \div 9)$

$\angle T \approx 47^\circ$

so $\angle S = 180^\circ - 110^\circ - 47^\circ = 23^\circ$

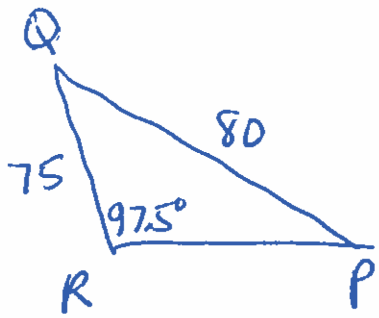
quick ✓

ex #3. Try on your own on p 802.

In $\triangle PQR$, $m\angle R = 97.5^\circ$, $r = 80$, and $p = 75$.

Find $m\angle P$.

$$\boxed{\frac{p}{\sin P} = \frac{r}{\sin R}}$$



$$\frac{75}{\sin P} = \frac{80}{\sin 97.5}$$

$\sin P = 75 * \sin(97.5) \div 80$

$P = \sin^{-1}(\text{ans})$

$\boxed{P = 68^\circ}$

Assignment:

p. 803–804

(1–5, 8, 11, 13, 17, 18)

✱ read directions

✱ must find all missing sides and angles.