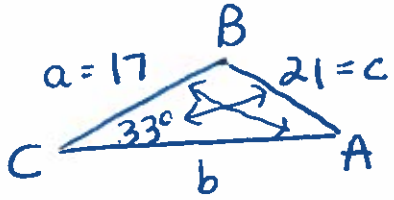
14-5 The Law of Cosines

Review: Use the Law of Sines

1. In $\triangle ABC$, $m\angle C = 33°$, $a = 17$ and $c = 21$. Find $m\angle A$.

1st: Draw pic



$$\frac{\text{side}}{\sin(\angle \text{across from that side})} = \frac{\text{side}}{\sin(\angle \text{across from that side})}$$

$$\frac{\text{side} \rightarrow c}{\sin C \uparrow \text{angle}} = \frac{a \leftarrow \text{side}}{\sin A \uparrow \text{angle}}$$

$$\frac{21}{\sin 33°} = \frac{17}{\sin A}$$

cross multiply ÷ by 21

$$\frac{21 \cdot \sin A}{21} = \frac{17 \cdot \sin 33}{21}$$

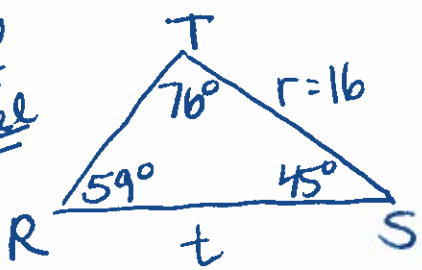
$$\angle A = \sin^{-1}\left(\frac{17 * \sin(33)}{21}\right)$$

$\angle A = 26°$

* IF you are finding an $\angle$ you have to do the inverse of sine ($\sin^{-1}$)

2. In $\triangle RST$, $m\angle R = 59°$, $m\angle S = 45°$, and $r = 16$. Find t. Since you know $\angle R + \angle S$, you can 1st find $m\angle T$ by $180° - 59° - 45° = 76°$ (180° in a $\triangle$)

1st: Draw pic + Label



set up proportion

$$\frac{t}{\sin T} = \frac{r}{\sin R}$$

(Don't use S because you don't know the side across from $\angle S$)

$$\frac{t}{\sin 76°} = \frac{16}{\sin 59°}$$

close the parentheses

$t = \sin(76) * 16 \div \sin(59)$ $\quad t = 18.1$

You can't use Law OF Sines on triangles where you only know all 3 sides or 2 sides and the ∠ between them.

So, we use Law of Cosines for SSS & SAS.

* Do not use Law OF Cosines for right △'s.

CASE #1: SSS

To Find an angle when given all 3 sides, use one of these 3 formulas. (SSS)

① $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

② $\cos B = \dfrac{a^2+c^2-b^2}{2ac}$

③ $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

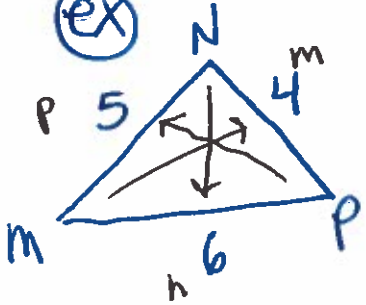
you will have to take $\cos^{-1}$(ans) to get the angle measure

notice that these are always the same letter

If the △ had different letters, like △XYZ then use the same concept...

$$\cos X = \frac{y^2+z^2-x^2}{2yz}$$

ex



Find m∠m.

$$\cos m = \frac{n^2+p^2-m^2}{2np}$$

$$\cos m = \frac{6^2+5^2-4^2}{2\cdot 6\cdot 5}$$

$$\cos m = \frac{3}{4} \qquad m = \cos^{-1}\left(\frac{3}{4}\right)$$

hint: use Fraction button (alpha Y=1) with newer calculators OR use parentheses $(6^2+5^2-4^2)\div(2*6*5)$ then take $\cos^{-1}$(ans)

TO Find another ∠ in the Δ, use the same concept.

Now let's Find $m\angle P$.

so.... $\cos P = \frac{m^2+n^2-p^2}{2 \cdot m \cdot n}$ ← these are all sides

always same letters

angle

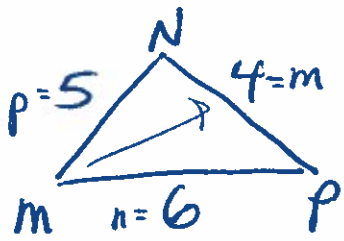
always the same

$\cos P = \frac{4^2+6^2-5^2}{2 \cdot 4 \cdot 6}$

$\cos P = \frac{9}{16}$

$P = \cos^{-1}\left(\frac{9}{16}\right)$

$P \cong 55.77$ OR 56°

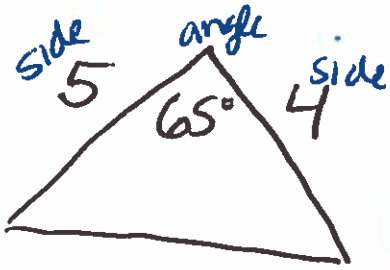


TO Find $m\angle N$, since we now know $m\angle M = 41^\circ$ and $m\angle P = 56^\circ$, $m\angle N = 180^\circ - 41^\circ - 56^\circ$

$m\angle N = 83^\circ$

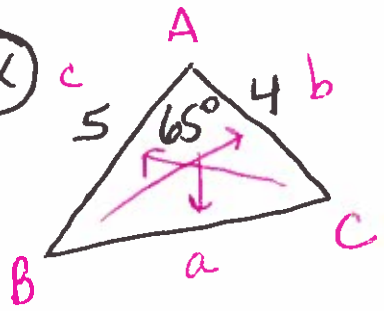
SAS: CASE #2 To Find a side, when you are given 2 sides and the ∠ between them use one of the following.

1. $a^2 = b^2 + c^2 - 2b \cdot c \cdot \cos A$
2. $b^2 = a^2 + c^2 - 2ac \cdot \cos B$
3. $c^2 = a^2 + b^2 - 2ab \cdot \cos C$



★ after you substitute in all the #'s to solve for the side, you will have to square root both sides

(ex)



Find measure of side a.

$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

(same Letter)

$a^2 = 4^2 + 5^2 - 2 \cdot 4 \cdot 5 \cdot \cos 65^\circ$

$\sqrt{a^2} = \sqrt{24.09526953}$

Square root both sides to Find a.

type in calculator

[2nd] [x^2] [2nd] [(-)]

and your screen will look like

$\sqrt{}$ Ans

$a \approx 4.9$

IF you want to Find m∠B. then you have to use

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

or type in Ans^2 if it's still on your screen

$\cos B = \dfrac{4.9^2 + 5^2 - 4^2}{2 \cdot 4.9 \cdot 5}$

$\cos B \approx .6742175111$

$B = \cos^{-1}(.6742175111)$

OR type in $\cos^{-1}$(ans)

$B \approx 47.60658 \approx 48^\circ$

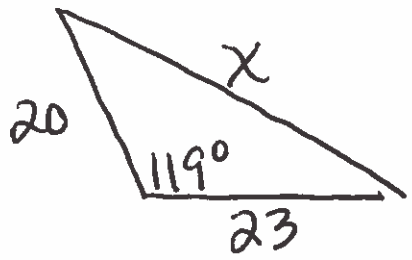
So IF $m\angle A = 65^\circ$ and $m\angle B = 48^\circ$, then

$m\angle C = 180 - 65 - 48 = 67^\circ$

* IF a problem asked you to Find all remaining sides + angles this is what you would

* Try these examples on your own Find X.
Find all answers to the nearest tenth.

ex



SAS case

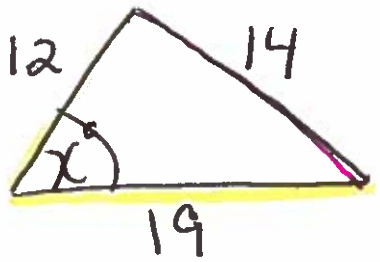
$x^2 = 20^2 + 23^2 - 2(20)(23)\cos(119°)$

$\sqrt{x^2} = \sqrt{1375.02485}$

then take square root of both sides to get X alone

$X = 37.1$

ex



sides that form ∠X ↙ ↓ ↙ side across from ∠X

$\cos X = \frac{12^2 + 19^2 - 14^2}{2(12)(19)}$

$\cos X = \frac{103}{152}$

$X = \cos^{-1}\left(\frac{103}{152}\right)$

$X \approx 47.34115765$

$X \approx 47.3°$ ← put degree symbol on angle measures

SSS case
- given 3 sides Find the ∠
- don't Forget to take inverse cosine OF your answer to get the ∠ measure

14.5 p.5