LESSON 31

Modeling Probability

Venn Diagrams and Geometric Probability

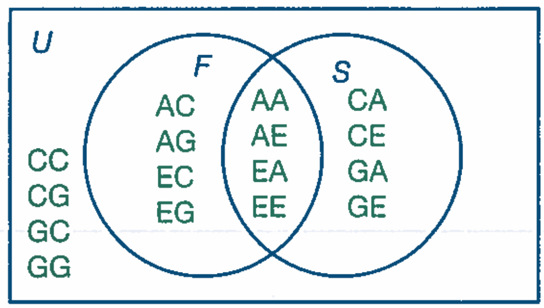
UNDERSTAND The probability of an event, A, occurring is represented as $P(A)$. Probability is expressed as a number from 0 to 1 that shows how likely the event is to occur. It can be written as a fraction, a decimal, or a percent and is given by the following ratio:

$$P(A) = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

So, for example, if you toss a number cube, there are 6 possible outcomes: {1, 2, 3, 4, 5, 6}. Suppose you want to know the probability of tossing an even number. In that case, there are 3 favorable outcomes: {2, 4, 6}.

$$P(\text{even}) = \frac{3}{6} = \frac{1}{2} \text{ or } 0.5 \text{ or } 50\%$$

UNDERSTAND **Joint probability** is the probability that two events will occur at the same time or one right after the other. For example, suppose you have two bags, each containing four cards lettered A, C, E, and G. Suppose you want to determine the probability of selecting vowels (A or E) from both bags. Placing the possible outcomes in a Venn diagram allows you to analyze them. Circle F represents selecting a vowel from the first bag. Circle S represents selecting a vowel from the second bag.

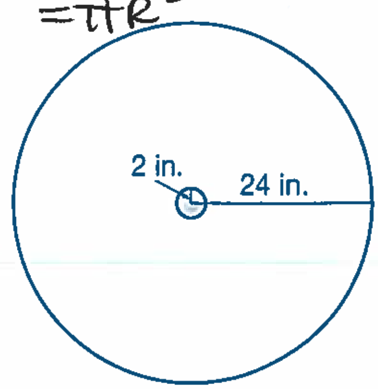


There are 16 possible outcomes. Of those, 8 involve selecting a vowel from the first bag, 8 involve selecting a vowel from the second bag, 4 involve selecting two vowels, and 4 involve selecting no vowels. Note that all outcomes are equally likely. The number of outcomes in a region divided by the total number of outcomes gives the probability for the event that the region represents.

There are 4 outcomes in the intersection of the sets, so the probability that you will select a vowel from both bags is $\frac{4}{16} = \frac{1}{4}$.

Area Circle $= \pi R^2$

UNDERSTAND For problems involving **geometric probability**, instead of counting the number of outcomes in a region, you find the total length, area, or volume of a region. For example, consider a target with a radius of 24 inches and a bull's-eye of radius 2 inches. The probability of hitting the bull's-eye when the target is hit is equal to the ratio of the area of the bull's-eye to the area of the target.



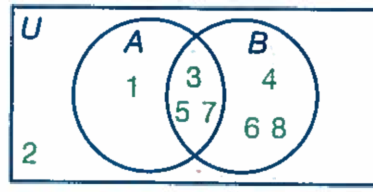
$$P(\text{bull's-eye}) = \frac{\pi(2)^2}{\pi(24)^2} = \frac{4\cancel{\pi}}{576\cancel{\pi}} = \frac{1}{144} \approx 0.0069$$

The geometric probability of hitting the bull's-eye is about 0.69%.

$A = \{1, 3, 5, 7\}$
$B = \{3, 4, 5, 6, 7, 8\}$

Connect

Lila is spinning a spinner with sectors numbered 1 to 8 and recording the results. Event A is spinning an odd number. Event B is spinning a number greater than 2. The Venn diagram shows the possible outcomes for this experiment.



Create a second Venn diagram to show the probabilities of the following:

$A - B$, $B - A$, $A \cap B$, and $\overline{A \cup B}$.

$A - B = \{1\}$ $P(A-B) = \frac{1}{8} = .125$ 12.5%

$B - A = \{4, 6, 8\}$ $P(B-A) = \frac{3}{8}$

$A \cap B = \{3, 5, 7\}$ $P(A \cap B) = \frac{3}{8}$

$A \cup B = \{1, 3, 4, 5, 6, 7, 8\}$ $P(A \cup B) = \frac{7}{8}$

$\overline{A \cup B} = \{2\}$

$P(\overline{A \cup B}) = \frac{1}{8}$

1 Find $P(A - B)$ and $P(B - A)$.

There are 8 possible outcomes:

1, 2, 3, 4, 5, 6, 7, and 8.

The region of circle A that does not overlap with circle B contains only one outcome: 1. So, $P(A - B) = \frac{1}{8} = 0.125$.

The region of circle B that does not overlap with circle A contains three outcomes: 4, 6, and 8. So, $P(B - A) = \frac{3}{8} = 0.375$.

2 Determine $P(A \cap B)$.

The region where circles A and B overlap shows their intersection.

There are three outcomes, 3, 5, and 7, in that region. So, $P(A \cap B) = \frac{3}{8} = 0.375$

3 Determine $P(\overline{A \cup B})$ and write the probabilities in a Venn diagram.

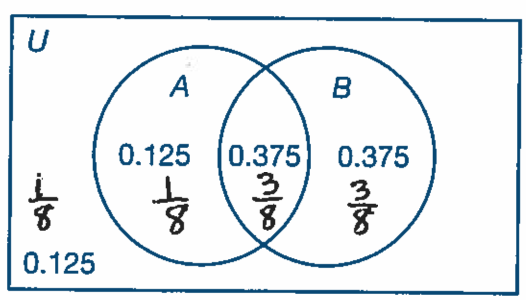
All of the outcomes inside circles A and B show the union of events A and B.

So, its complement is the one outcome, 2, that is outside the circles.

▶ $P(\overline{A \cup B}) = \frac{1}{8} = 0.125$

4 Draw the Venn diagram of probabilities.

Write the probabilities in the corresponding regions.



CHECK

Add together all four probabilities. What is the sum? Why? $= 1$

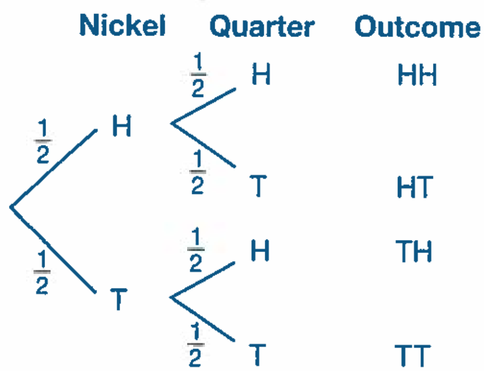
100% of the elements in the set

Tree Diagrams and Two-Way Tables

UNDERSTAND Suppose you toss two fair coins—a nickel and a quarter—at the same time. Each coin can land on either heads (H) or tails (T). So for each individual coin toss, the probabilities are:

$P(\text{H}) = \frac{1}{2}$ $\quad\quad$ $P(\text{T}) = \frac{1}{2}$

However, the probability of both coins landing on heads, $P(\text{HH})$, is not $\frac{1}{2}$. Drawing a **tree diagram** to represent all the possible outcomes can help you see this. The last column of the tree diagram below shows that there are 4 possible outcomes: {HH, HT, TH, TT}. All outcomes are equally likely, so the probability of each outcome is $\frac{1}{4}$. That means $P(\text{HH}) = \frac{1}{4}$.



UNDERSTAND Probabilities can also help you understand data that are collected by surveying a representative **sample** of people drawn from a larger **population**. If you want to compare two categorical variables such as gender and reading habits, you can construct a **two-way frequency table** like the one below. The table shows **joint frequencies** and **marginal frequencies**.

	Reads for Pleasure	Reads Only for School	Total
Boys	20	30	50
Girls	30	20	50
Total	50	50	100

$\frac{20}{50} = 40\%$

Joint frequencies are in the body of the table.

Marginal frequencies are in the "Total" row and "Total" column.

If you divide a particular frequency by one of the totals, you can determine its **relative frequency** by row, by column, or for the entire table. For example, $\frac{20}{50}$, or 40%, of all the boys surveyed said they like to read for pleasure, and $\frac{20}{100}$, or 20%, of all the people surveyed above were boys who said they like to read for pleasure. This means that if you select a boy from the population at random, there would be a 40% chance that he reads for pleasure, and if you select a person from the population at random, there would be a 20% chance that that person would be a boy who reads for pleasure

Connect

The table shows the results of a survey of students in grades 9, 10, 11, and 12 that asked them if they preferred rock music or rap music.

What is the chance that a student chosen from the school at random is a 10th-grade student who prefers rap music to rock music?

	Rock	Rap
9th Grade	15	35
10th Grade	26	24
11th Grade	25	25
12th Grade	32	18

1 Extend the table.

The table above shows only joint frequencies. Add the numbers in the columns and rows to find the marginal frequencies.

	Rock	Rap	Total
9th Grade	15	35	50
10th Grade	26	24	50
11th Grade	25	25	50
12th Grade	32	18	50
Total	98	102	200

2 Find the cell (joint frequency) and total (marginal frequency) that you need.

Look at the cell in the table that lies in the 10th-grade row and the rap column. That cell contains the value 24.

The question asks for the probability of choosing a 10th grader who prefers rap from among all the students at the school. So, use the total for the entire table, 200.

3 Calculate the probability.

$$P(\text{10th grader who prefers rap}) = \frac{\text{number of 10th graders who prefer rap}}{\text{total number of students}} = \frac{24}{200} = 0.12$$

▶ If a student is selected from the school at random, there is a 12% chance that the student will be a 10th-grade student who prefers rap.

TRY

What is the chance that a student chosen from the school at random prefers rock music?